

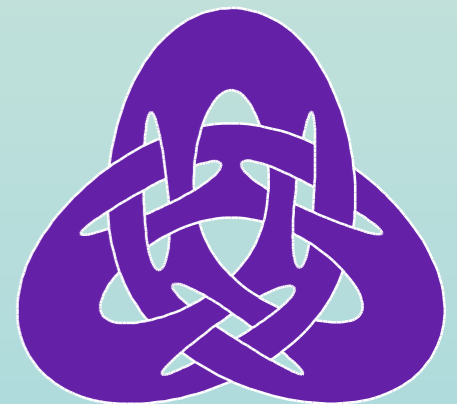
# *Druid*

## Representation of Interwoven Surfaces in 2½D Drawing

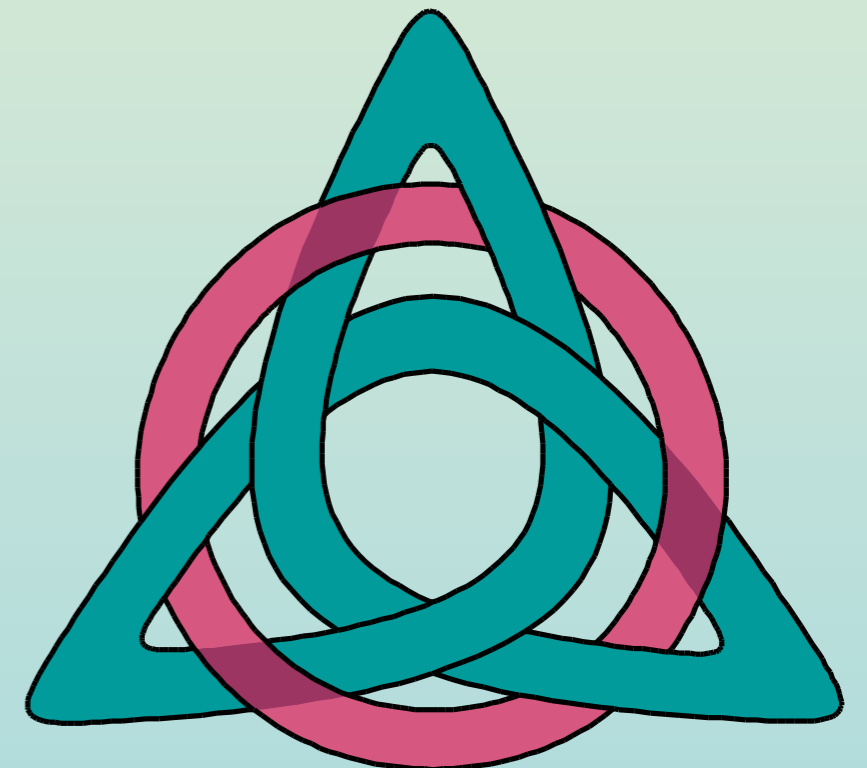
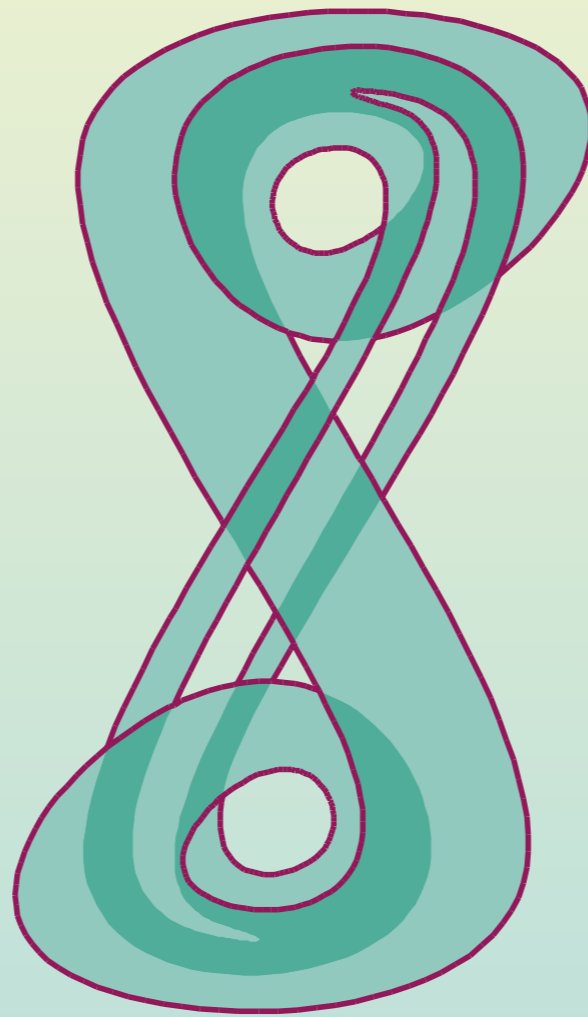
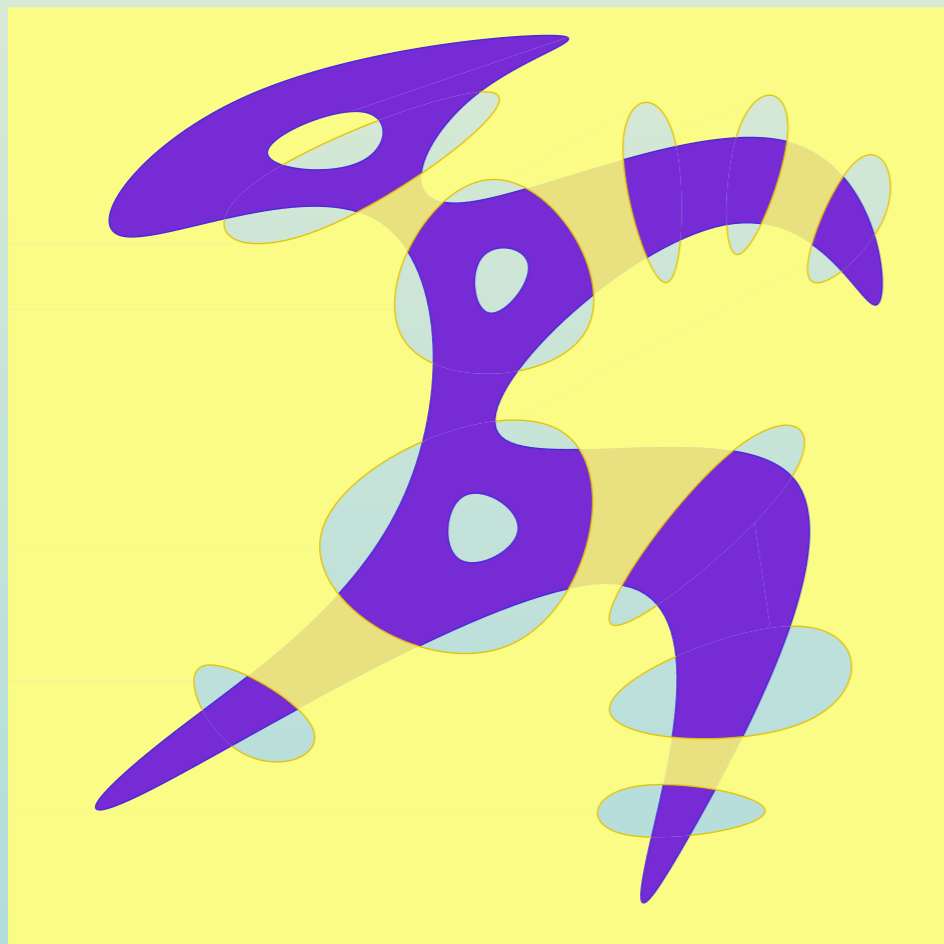
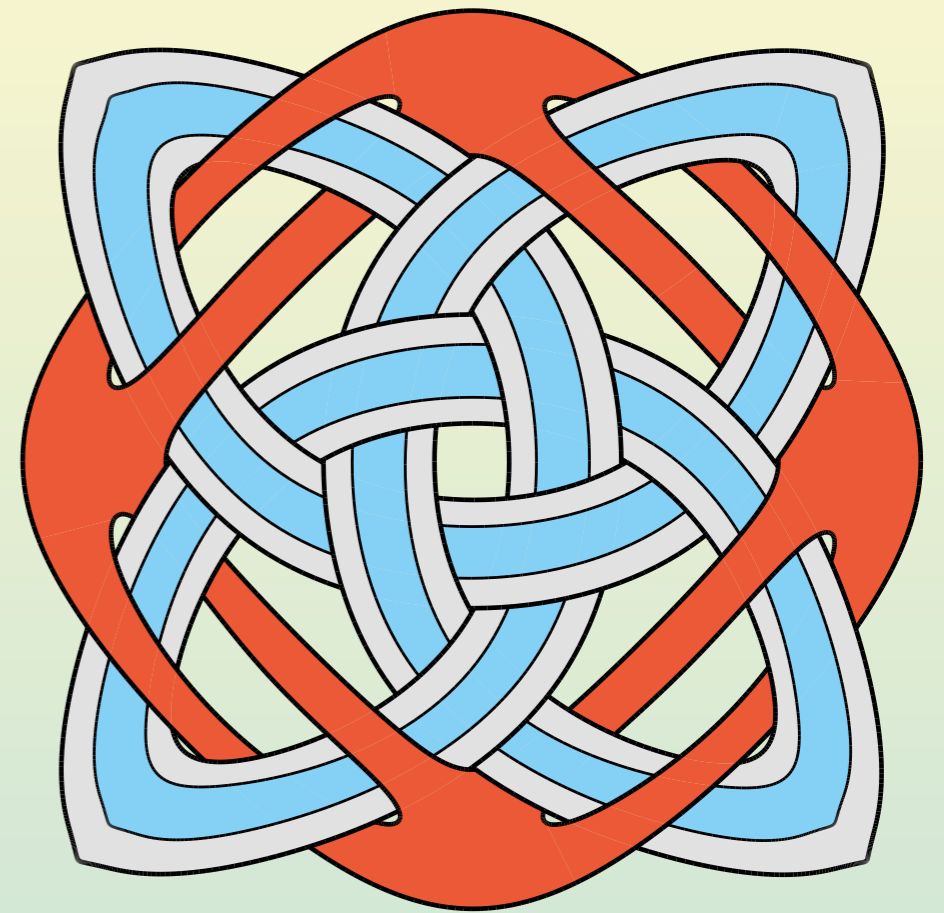
Keith Wiley

Thesis advisor: Lance R. Williams

University of New Mexico  
Department of Computer Science  
Albuquerque, NM 87131 USA



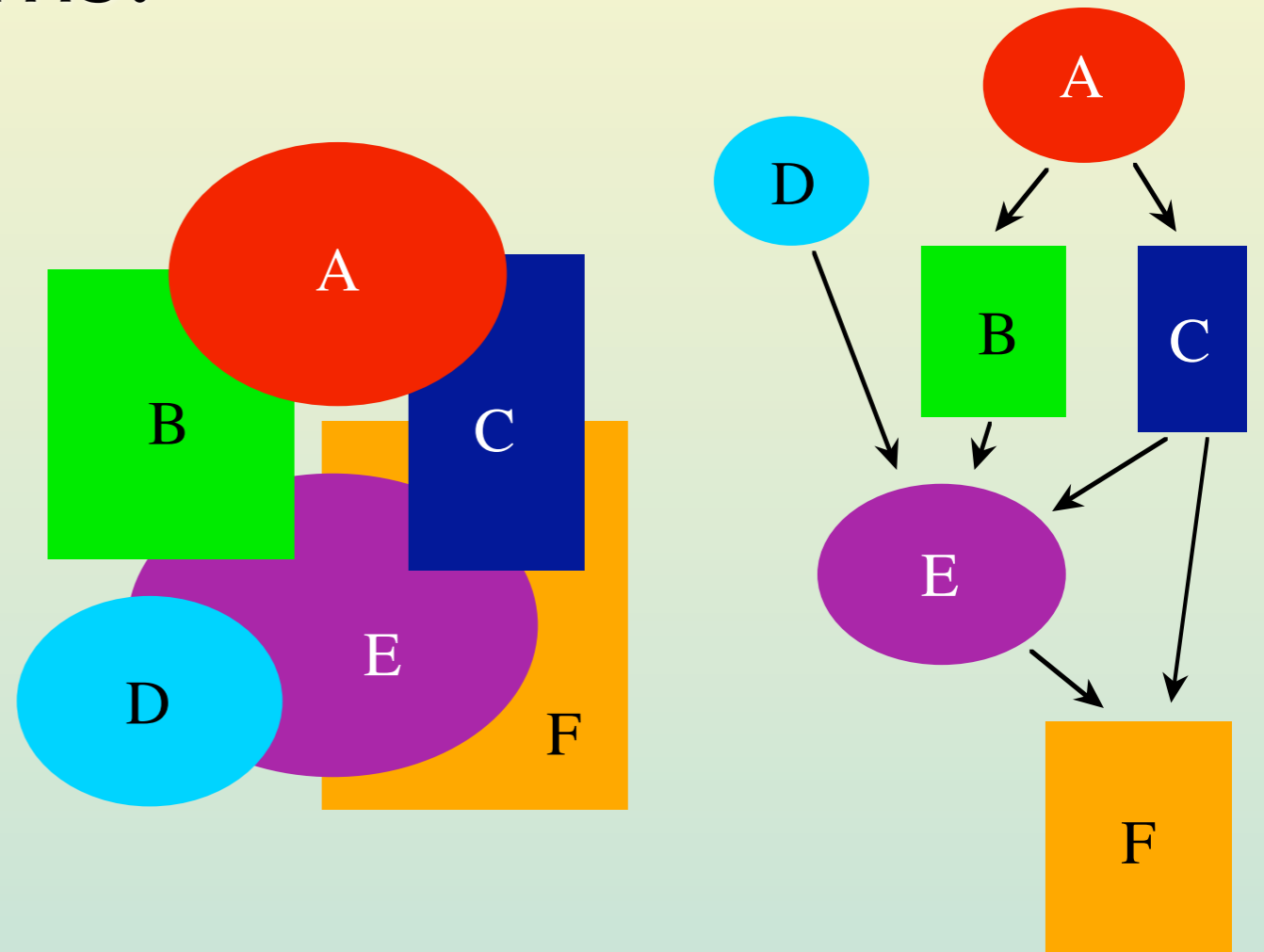
# Interwoven 2<sup>1</sup>/<sub>2</sub>D Scenes



# Introduction

Existing drawing programs:

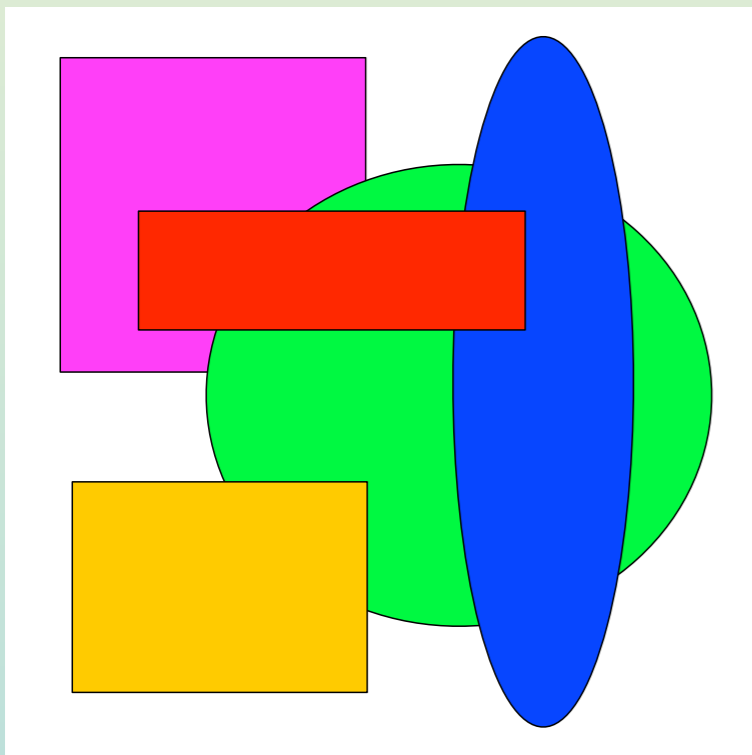
- Use distinct layers
- Impose a DAG
- Do not permit interwoven surfaces



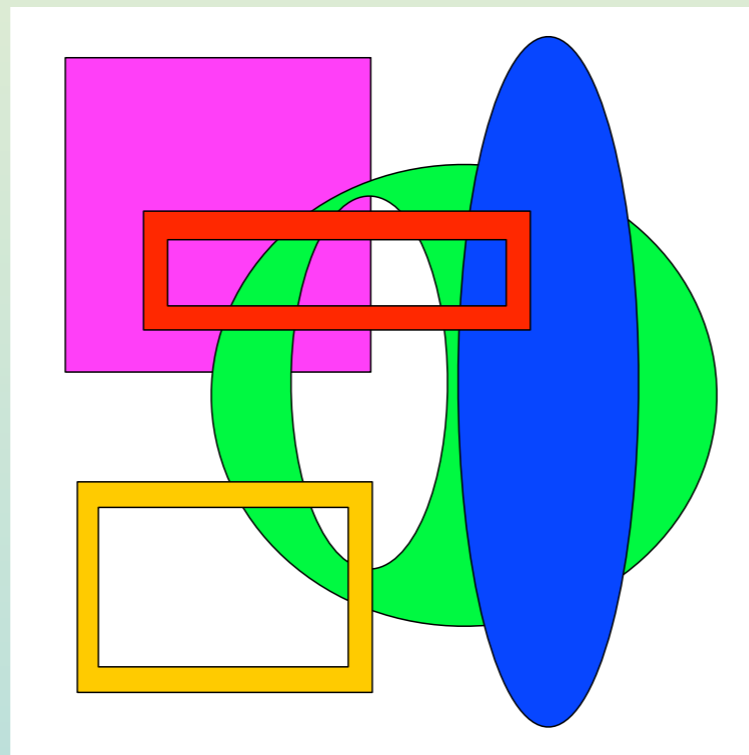
Our program, *Druid*, does not suffer from these limitations.

# Existing Drawing Programs

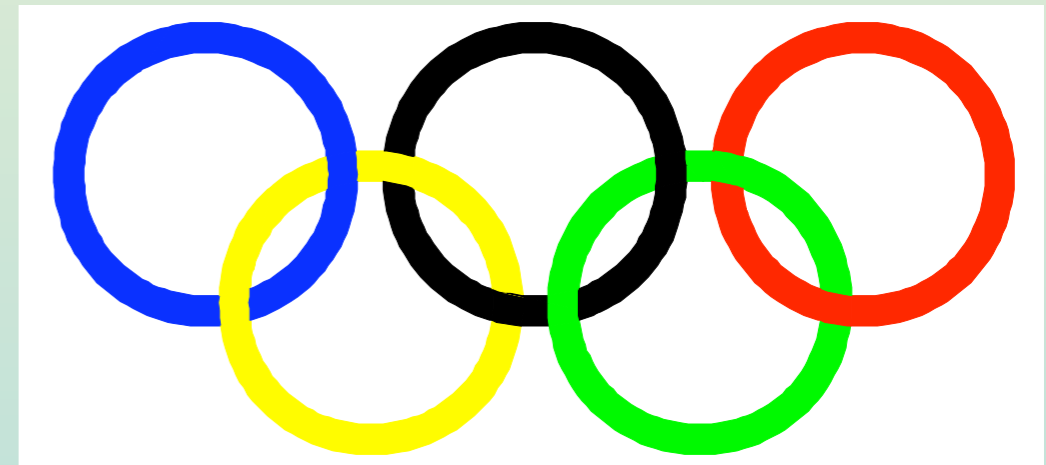
Noninterwoven  
layers



Boolean  
combinations of  
boundaries, *i.e.*,  
holes.

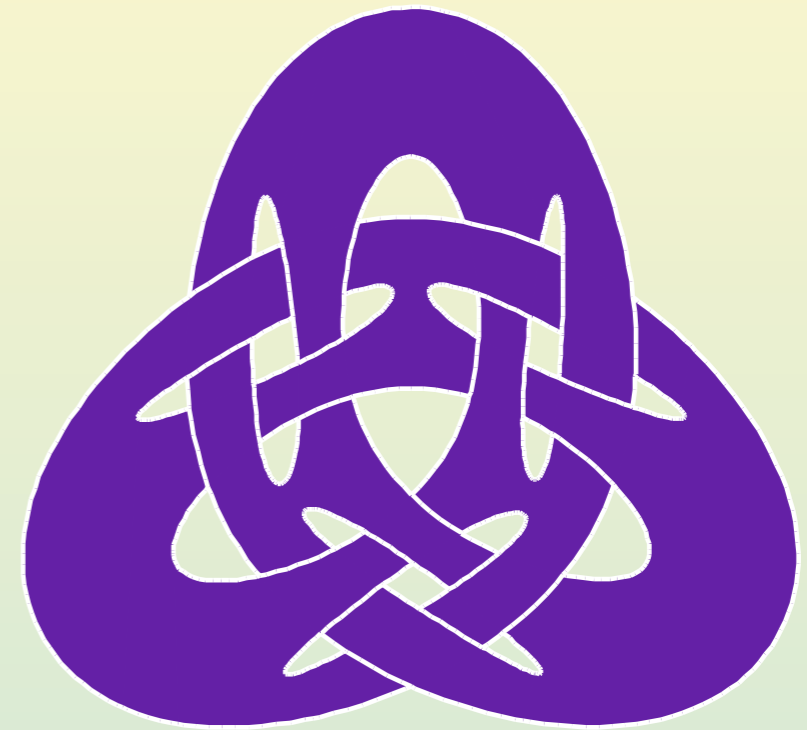
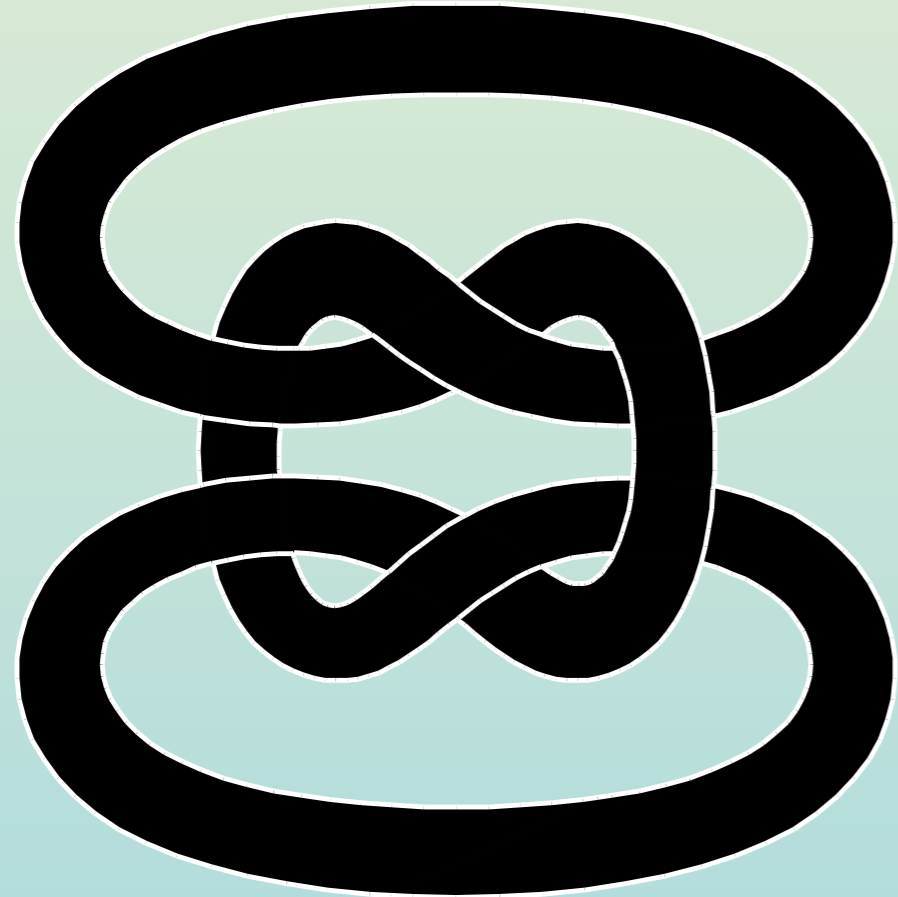
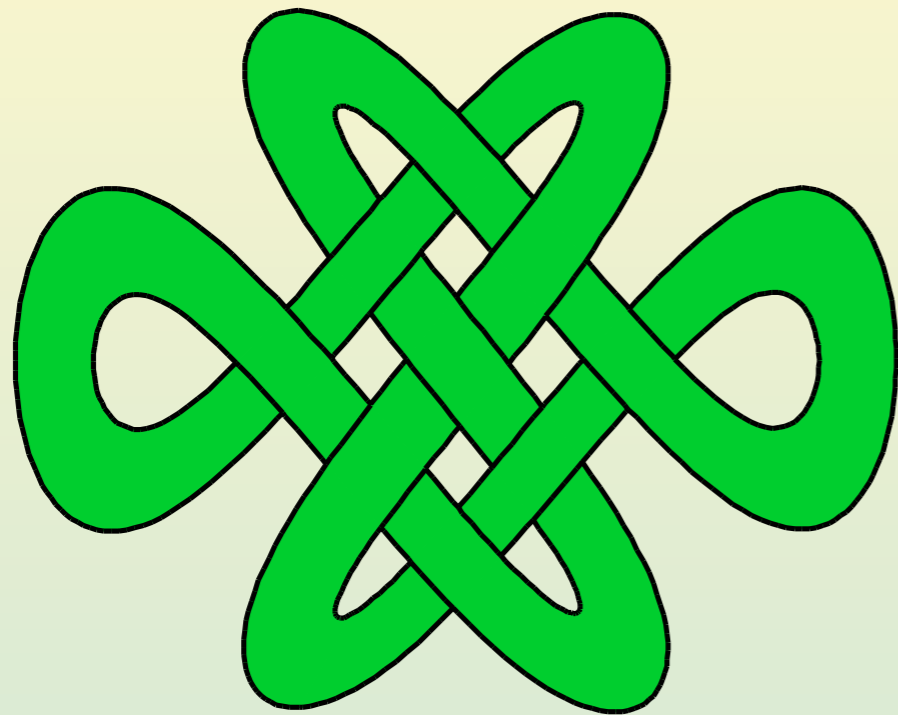


Do not span the full space  
of  $2^{1/2}D$  scenes.





# Knots vs. Interwoven Surfaces

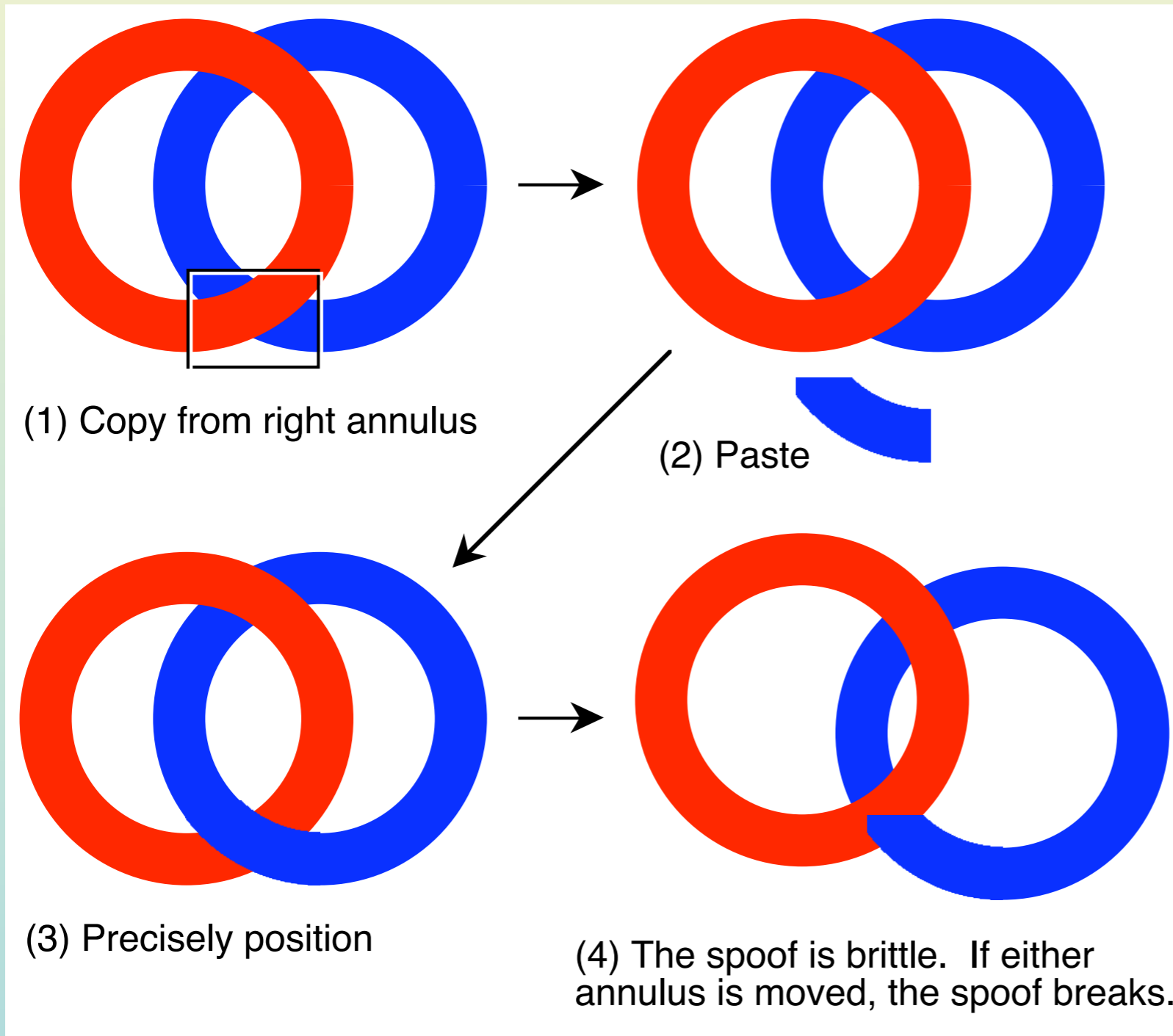


# Interwoven Surfaces in Conventional Drawing Programs

1. Spoofs
2. Painting planarized graphs,  
*e.g., Adobe Illustrator*
3. Local DAG manipulation,  
*e.g., MediaChance Real-Draw*

# Spoofs

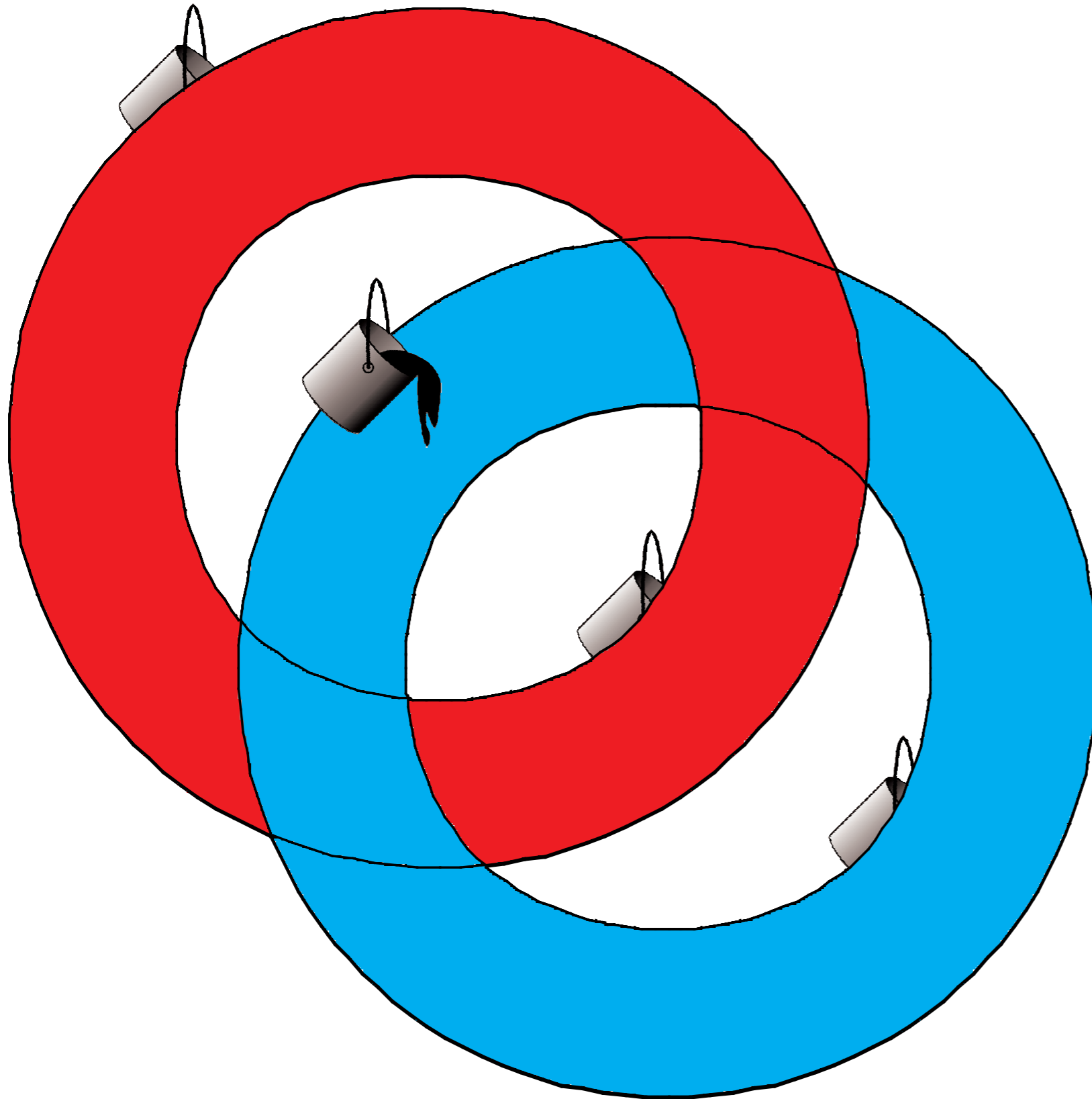
A layered arrangement that produces the illusion of interwoven surfaces



- Tedious to construct

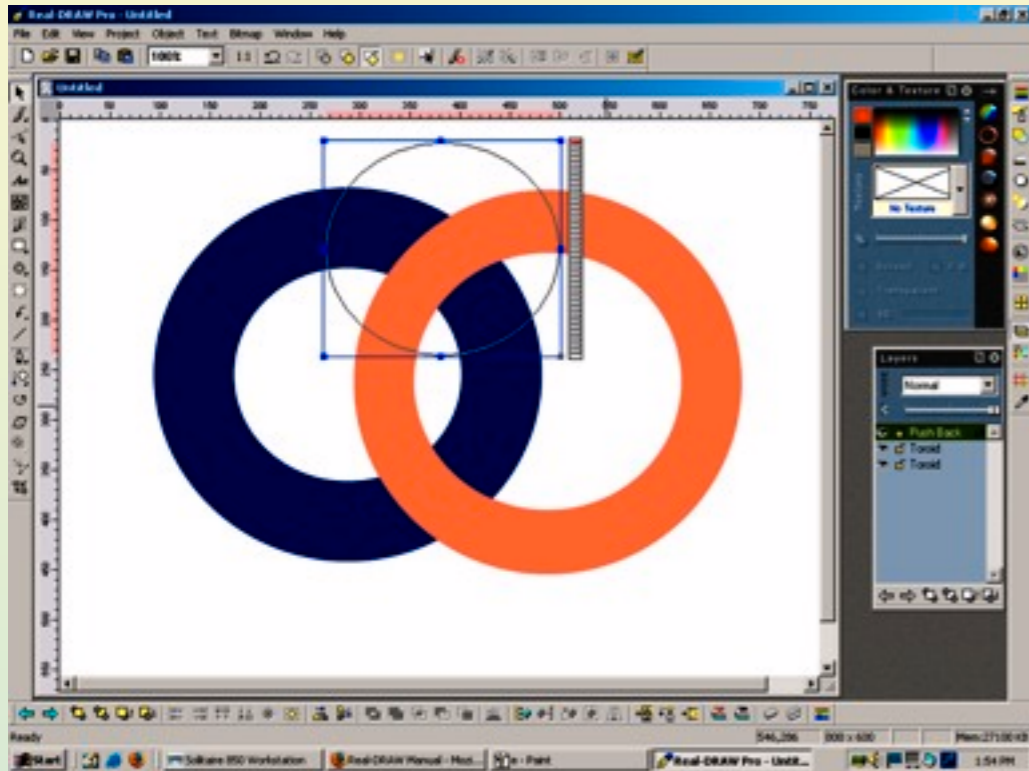
- Tedious to maintain

# *Adobe Illustrator Method*

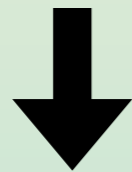


- Convert drawing to planar graph
- Paint faces of the graph independently

# MediaChance Real-Draw Pro-3

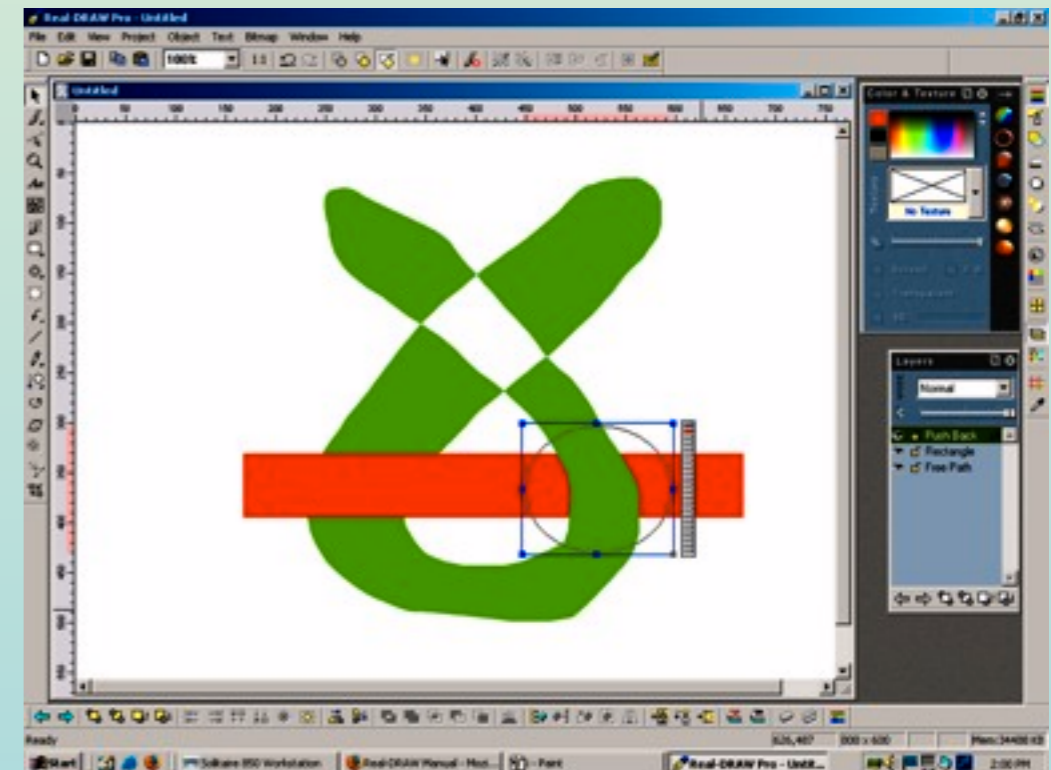
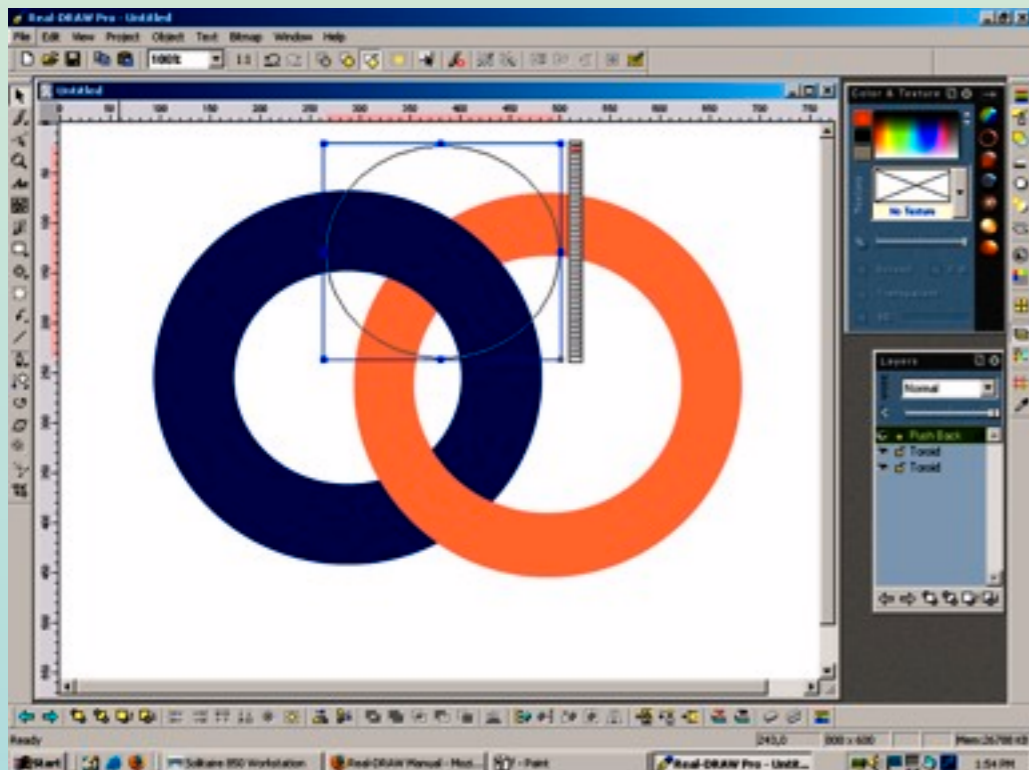


The right annulus is pushed down



**Push-back tool:** The user can push the top layer down (figures left)

- Insufficient for transparent surfaces
- Cannot represent self-overlapping surfaces (figure below)





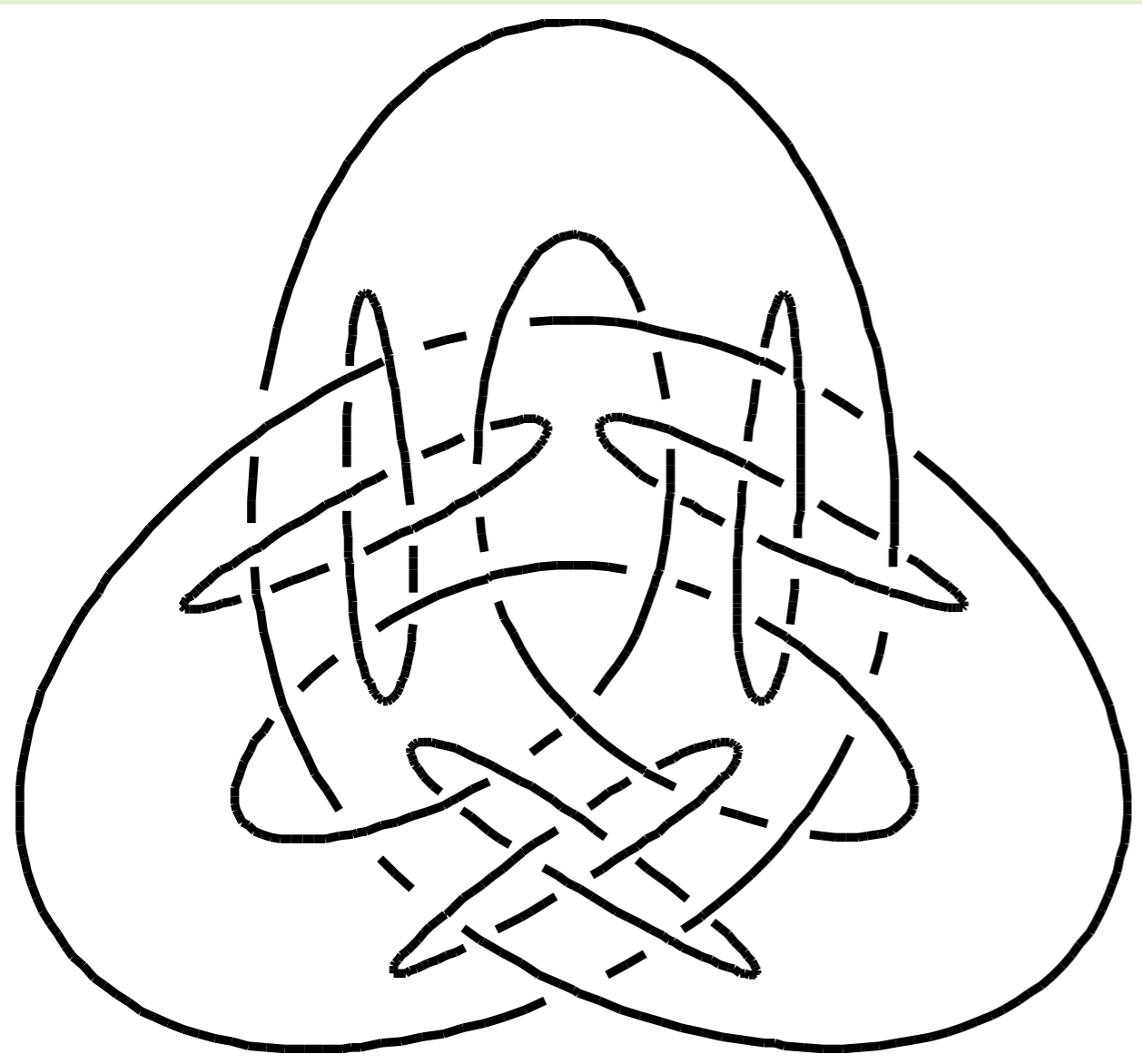
# Affordances

- **Feasibility** is not the sole issue. **Convenience** and **naturalness** are also issues.
- **Affordances**: The set of interactions that a physical object suggests for itself (Norman '02).
- Unlike conventional drawing programs, *Druid's* affordances are isomorphic to those of idealized physical surfaces.
- The user's experience is of interacting with surfaces, not pictures of surfaces.

# Druid's Representation

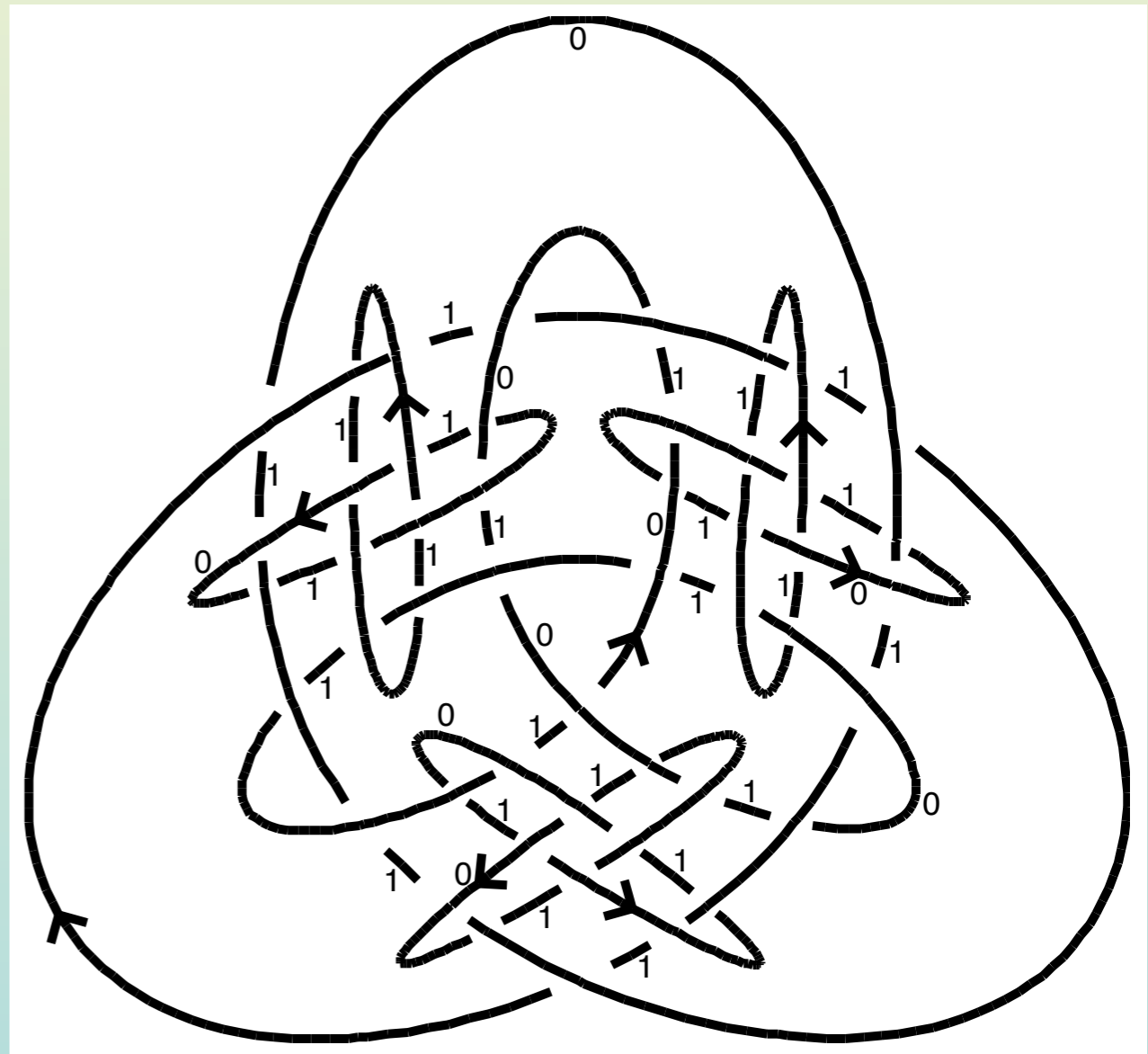
## ***Knot-diagram:***

A projection of closed curves indicating which curve is above where two cross



## ***Labeled knot-diagram*** (Williams '94):

***Sign of occlusion*** for every boundary (arrows)  
***Depth index*** for every boundary segment

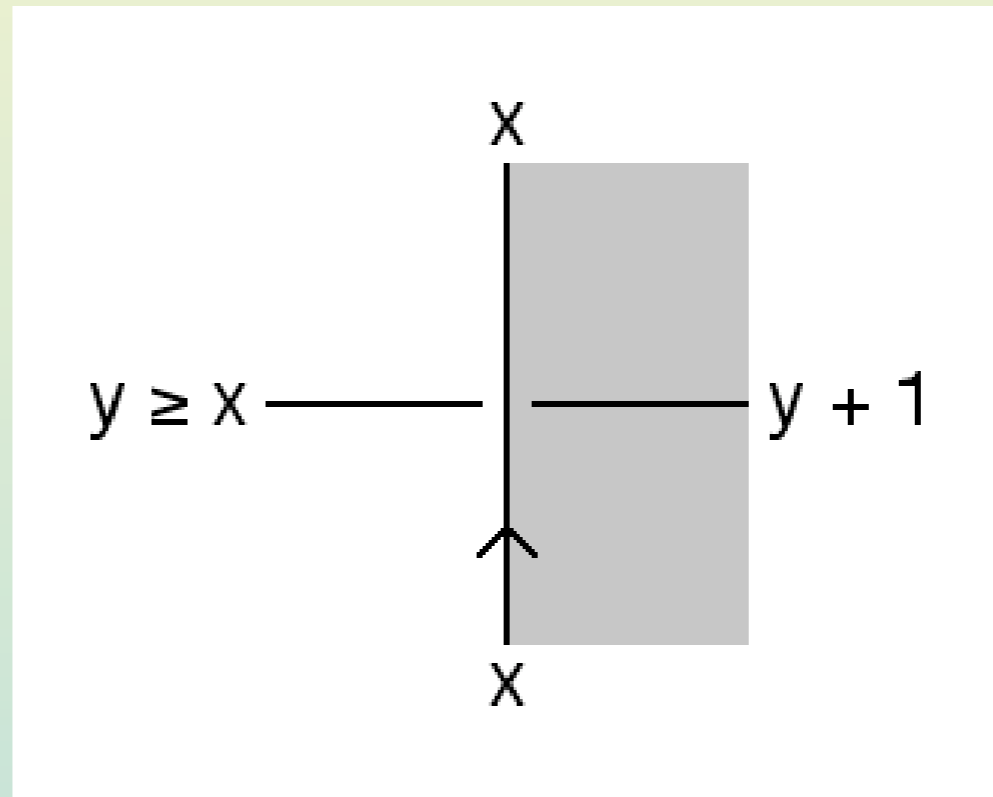




# Labeling Scheme

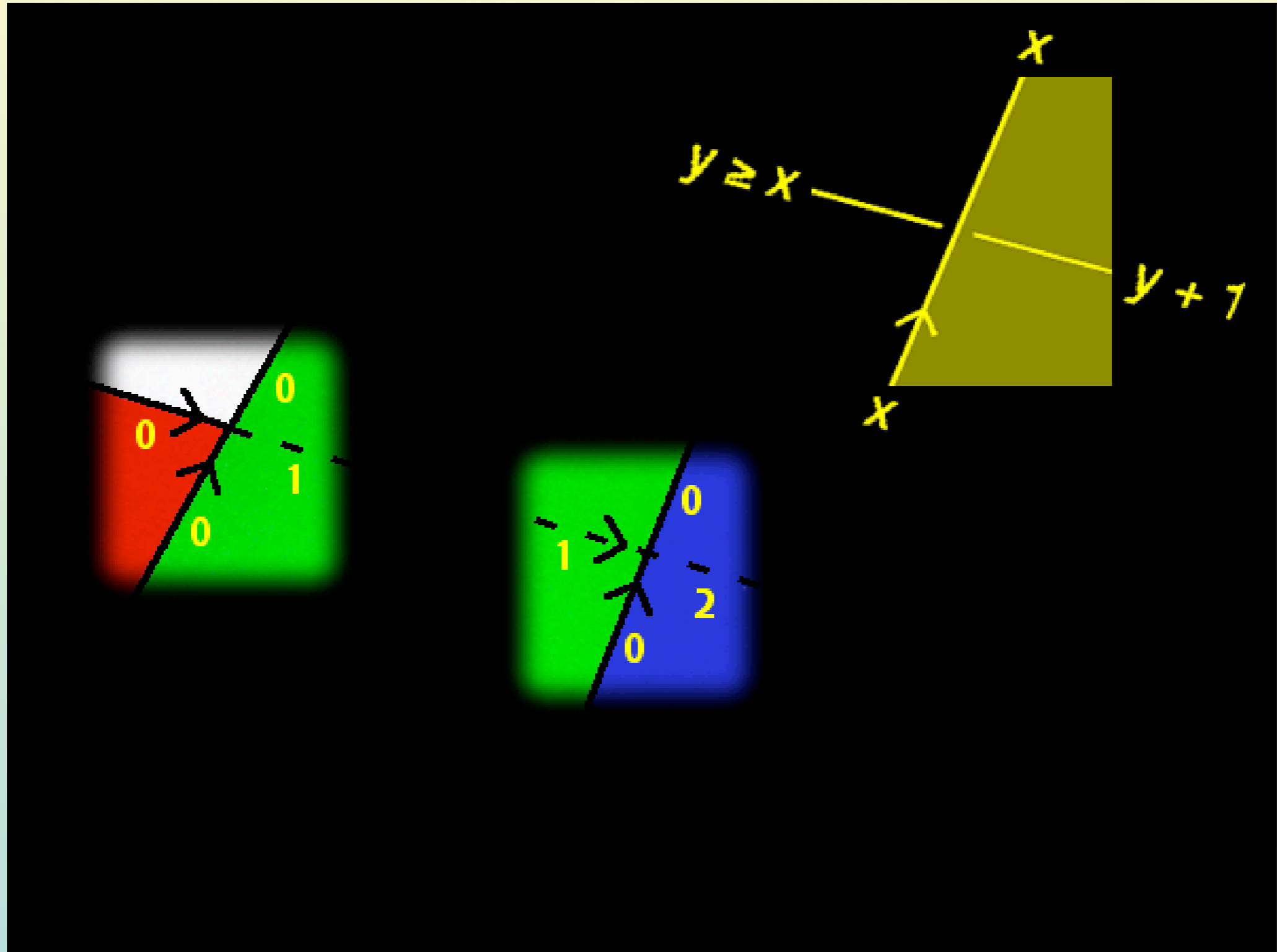
Imposes local constraints on the four boundary segment depths at a crossing

$x, y$ : boundary  
segment depths



***Legal labeling***: A labeling in which every crossing satisfies the ***labeling scheme***  
(Williams '94)

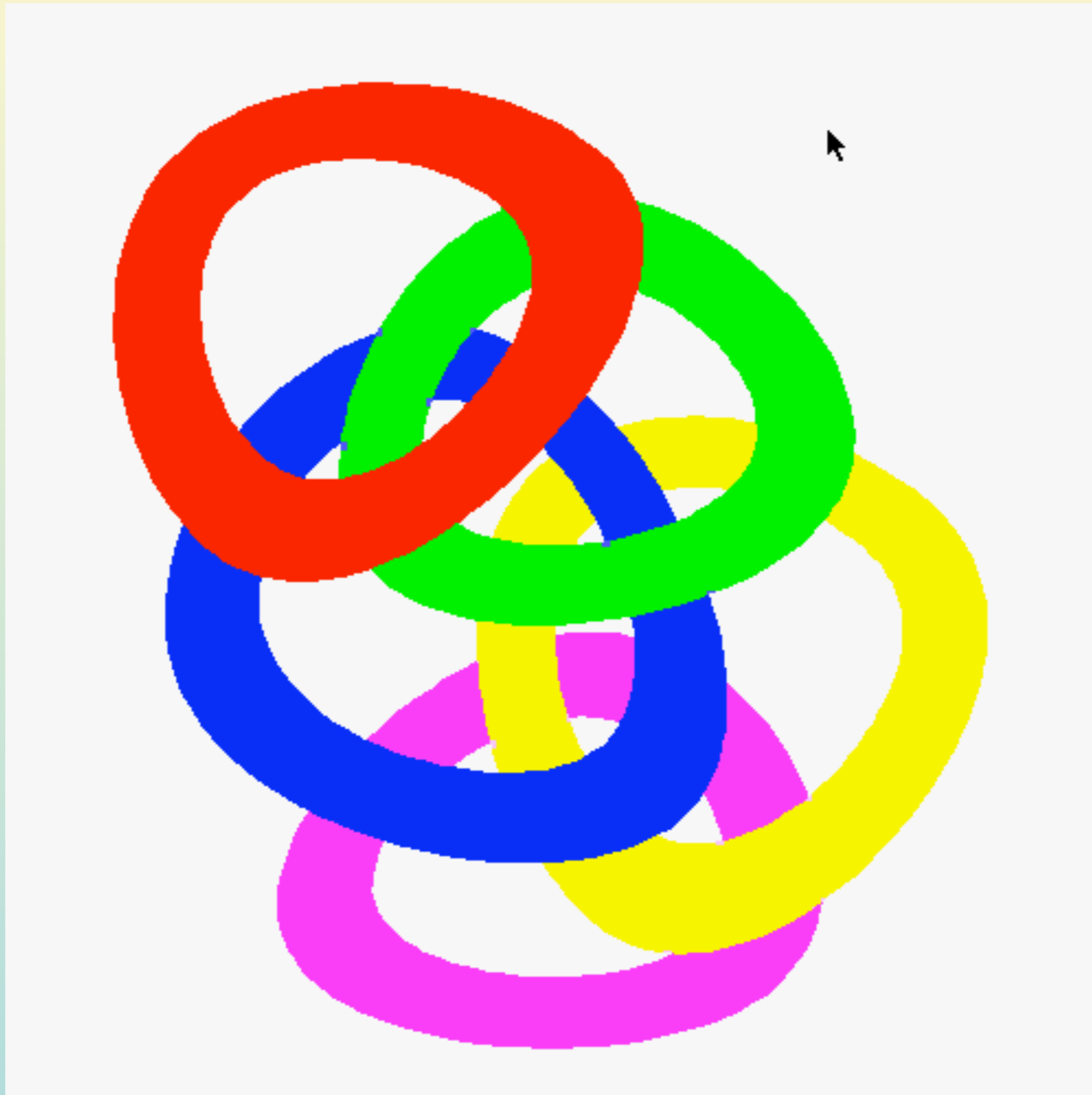
# Labeling Scheme Justification



# Using *Druid*



# The *Crossing-Flip* Interaction



# Drawing Program Interactions

- Create & delete boundaries
- Reshape & drag boundaries
- Crossing flip (Invert two surfaces' relative depths in an area of overlap)
- Sign-of-occlusion flip

# Effects of Interactions on the Labeling

## Requiring relabeling (topological change)

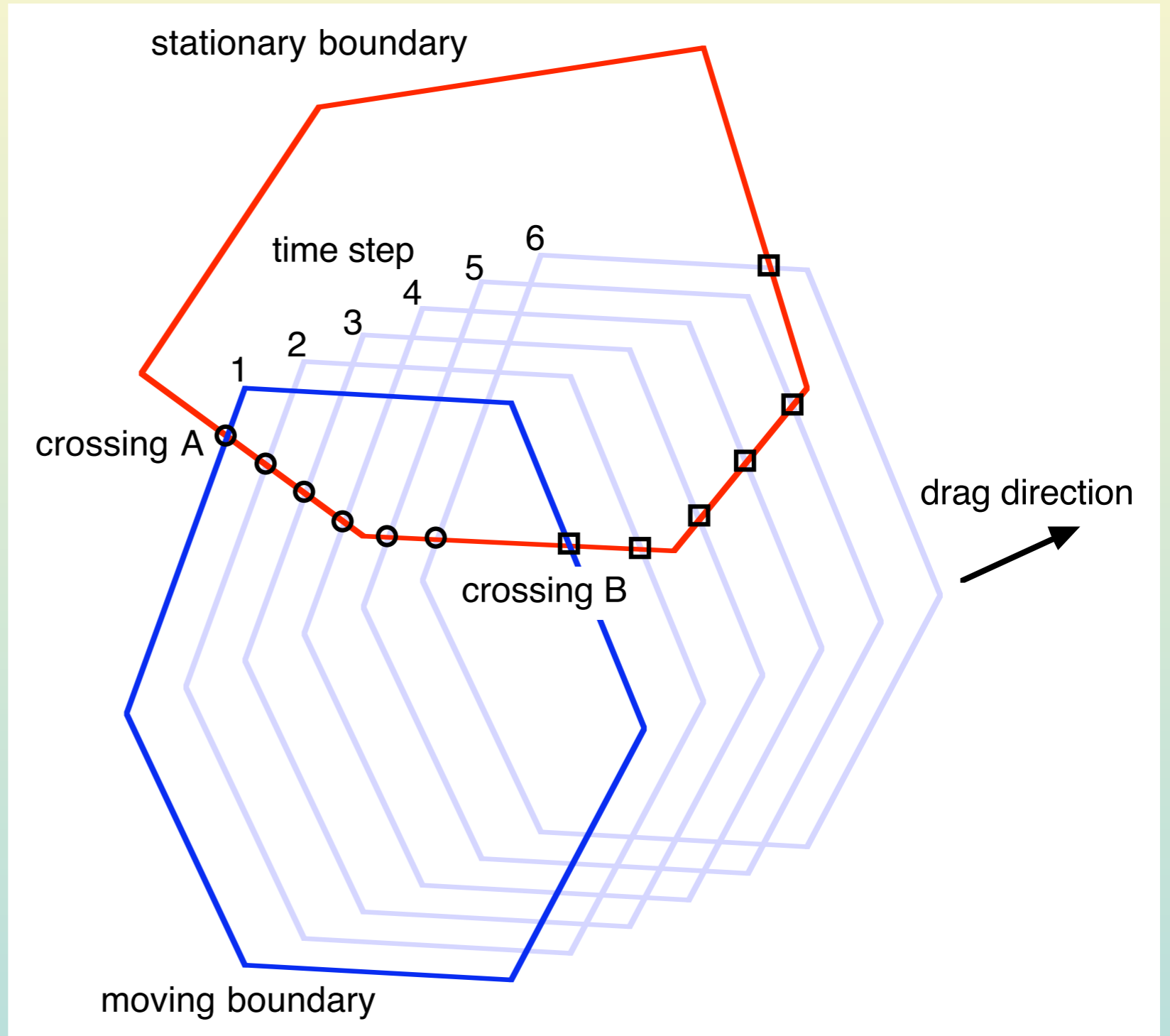
- Creation & deletion of crossings
- Reordering of crossings around boundaries
- Crossing-state flips
- Sign-of-occlusion flips

## Not requiring relabeling (no topological change)

- Reshaping or dragging boundaries without causing topological changes

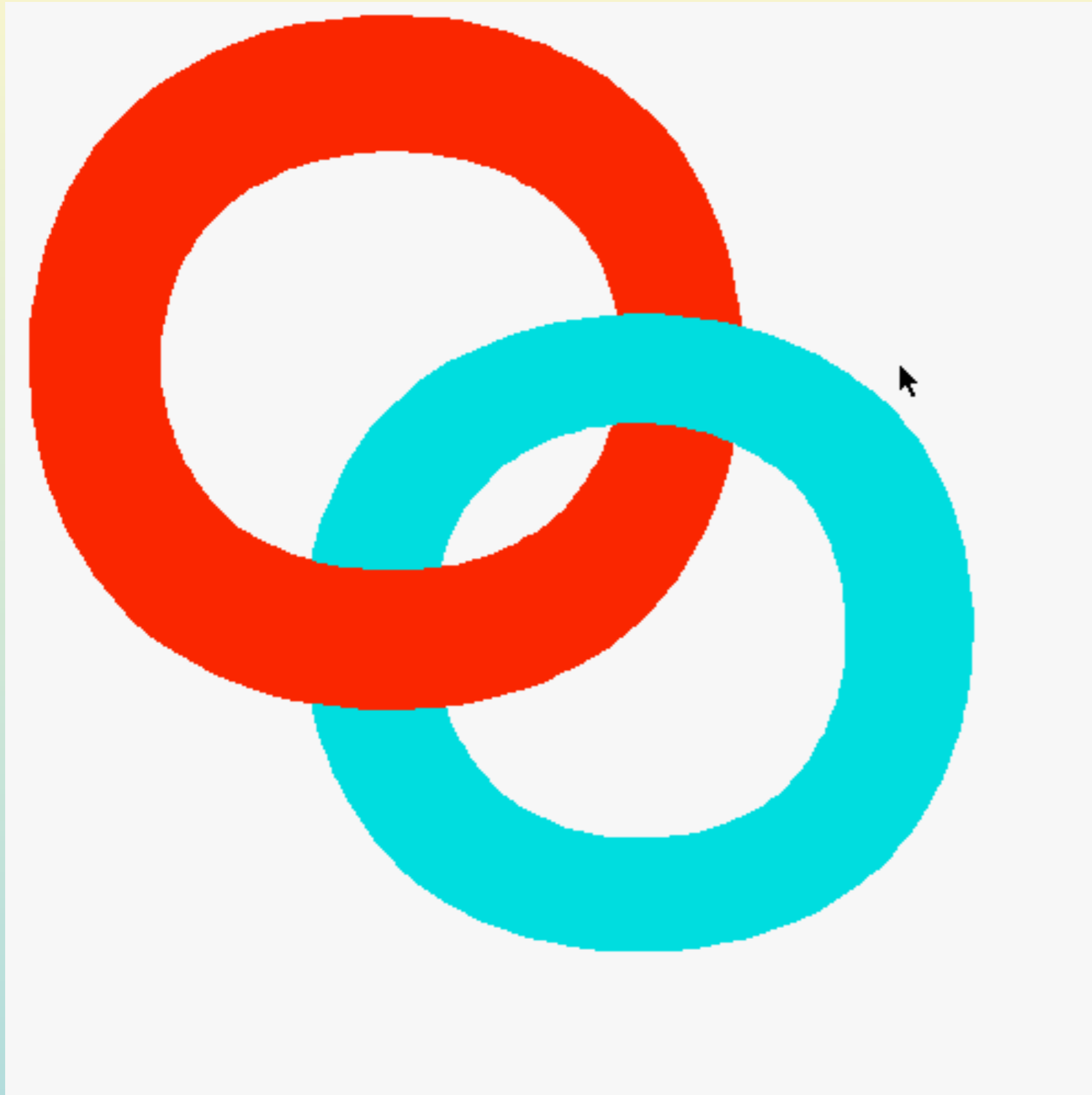
# Crossing Projection

- Important to preserve crossing-states
- Naive destruction/rediscovery of crossings would lose crossing-states
- *Druid* projects crossings as they move around boundaries





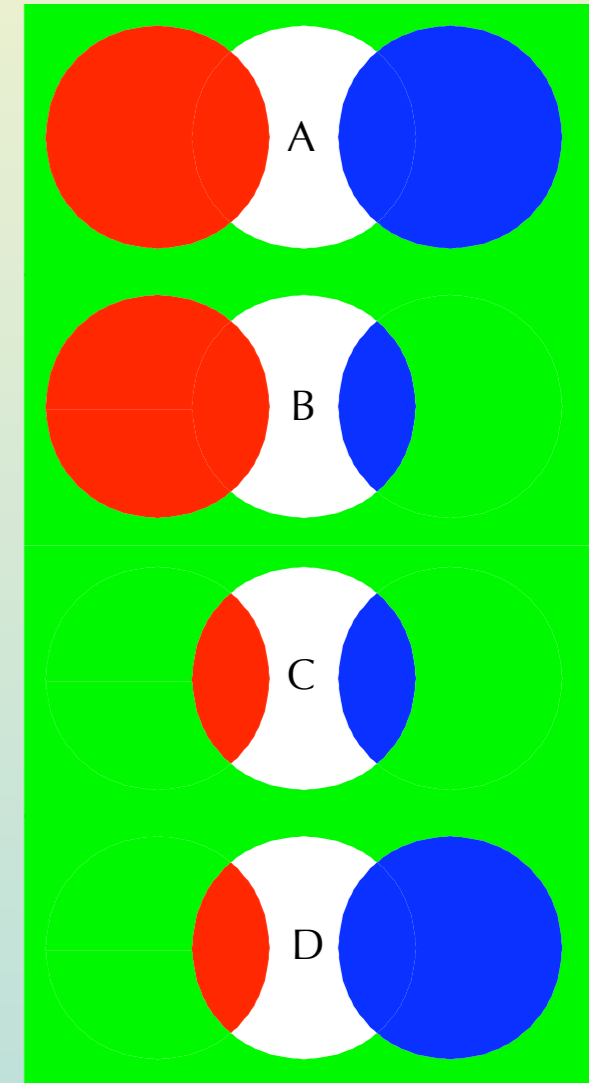
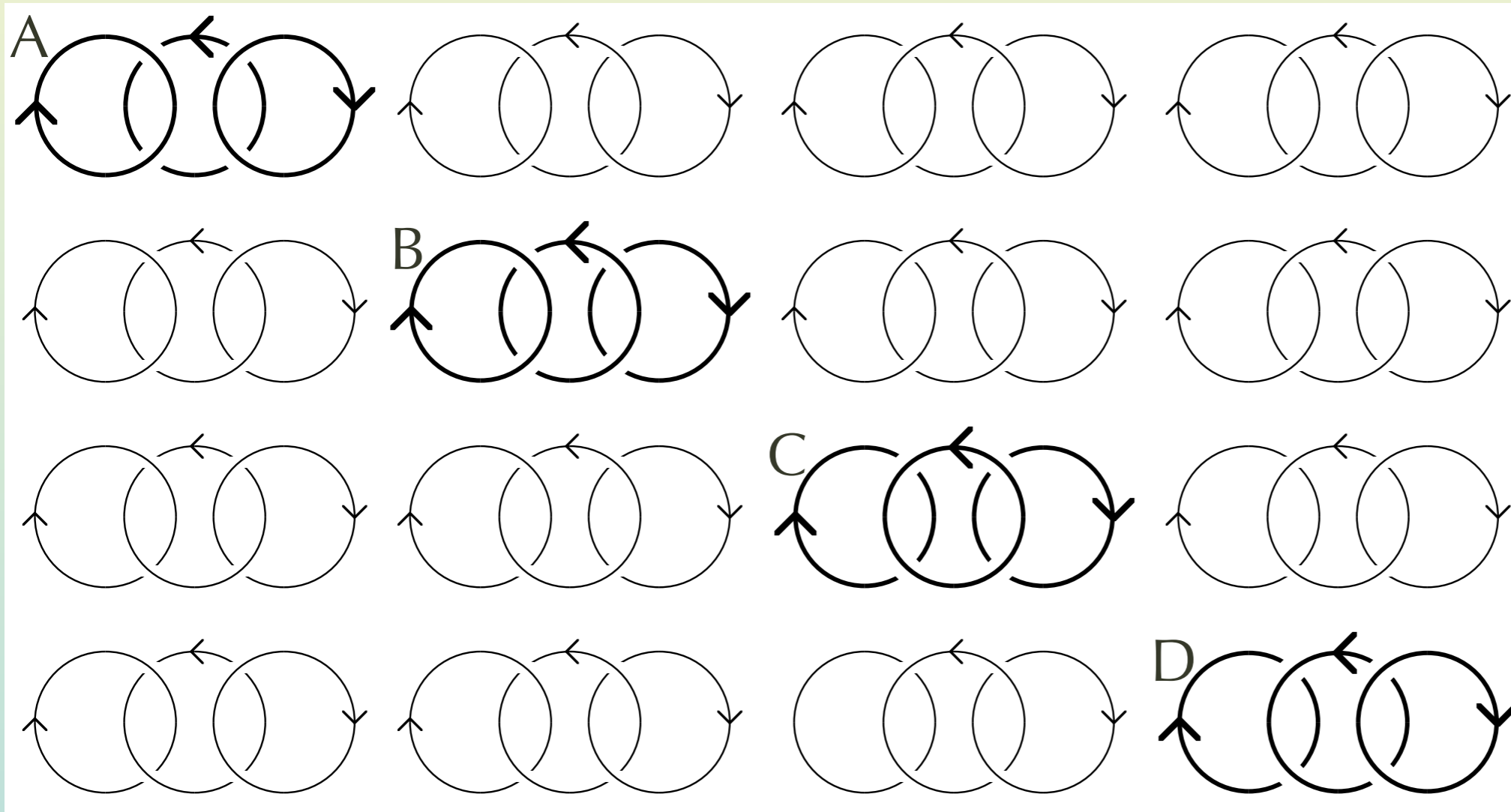
# Demonstration of *Druid*



- *Druid* knows to move both boundaries at once.
- *Druid* relabels when the interlock breaks.

# Finding a Legal Labeling

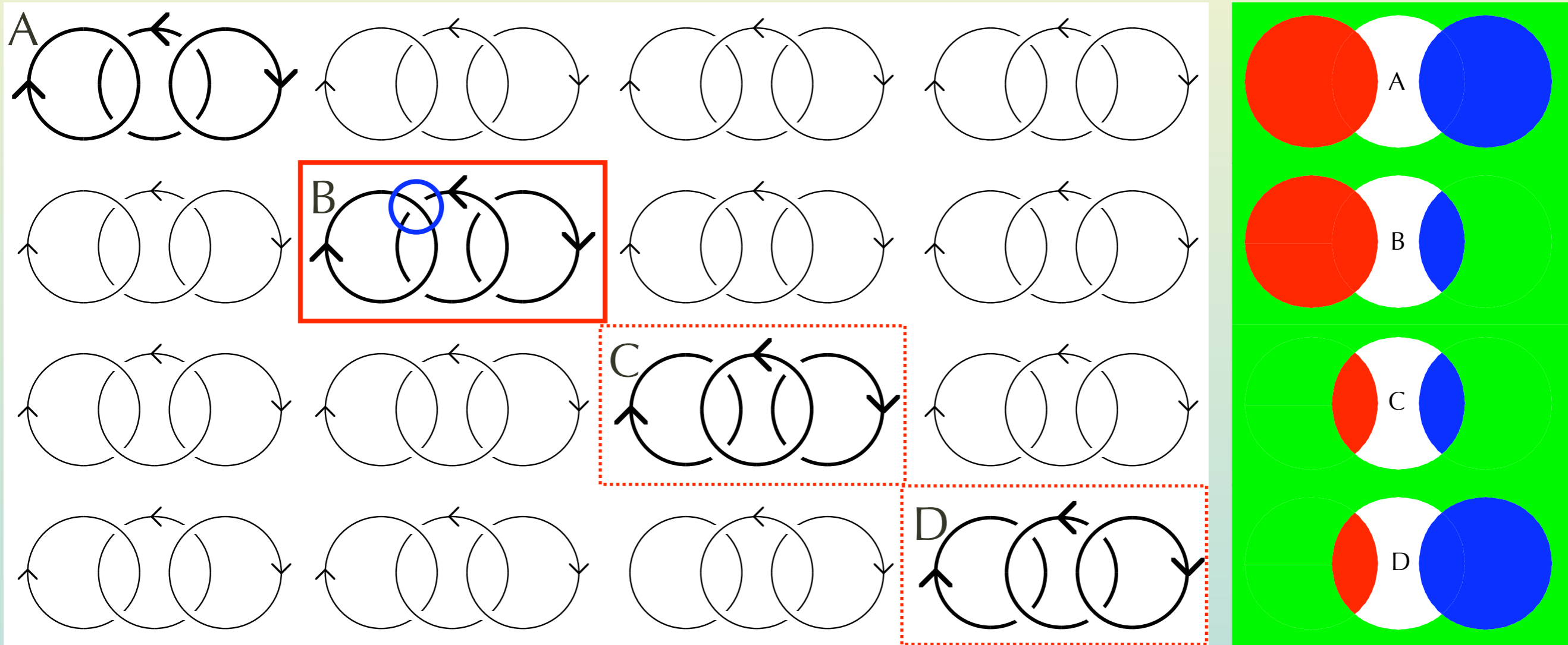
**Labeling space:** Possible labelings for a labeled knot-diagram. Labeling space size:  $2^c$



*Druid* maintains a legal labeling automatically.

# Minimum-Difference Search

*Druid* searches the *labeling space* for the *minimum-difference labeling*.



- Labeling is currently in state *B*.
- User clicks the **blue-circle** marked crossing.
- *C* and *D* are possible solutions, *C* is minimum difference from *B*.

# The Labeling Search

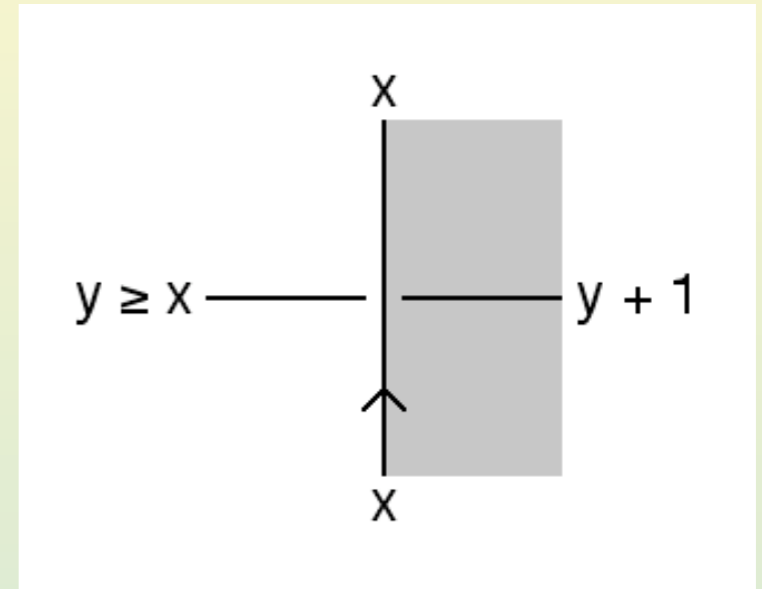
- Branch-and-bound
- Constraint propagation
- Iterative deepening
- Timeouts

# Branch-and-bound

- Search goal: *minimum difference labeling*
- Node expansion can never decrease the accumulated labeling difference
- Minimum difference legal solution gives the bound
- Search is truncated when the accumulated current difference exceeds the bound

# Constraint Propagation (Waltz '75)

- Orders the search so that legal solutions are found earlier
- Legal solutions define bounds
- Constraint propagation works in concert with branch-and-bound to increase search efficiency





# Iterative Deepening

- Branch-and-bound works best if good solutions are found earlier
- In good solutions, changes are localized to the *area of interest*
- Search is restarted with increasing *search horizons*



# Timeouts

- The search can take too long
- Two timeouts:
  - ***Very short timeout (0.1 sec)***: If a solution has been found during the search
  - ***Longer timeout (5.0 sec)***: If no solution has been found yet

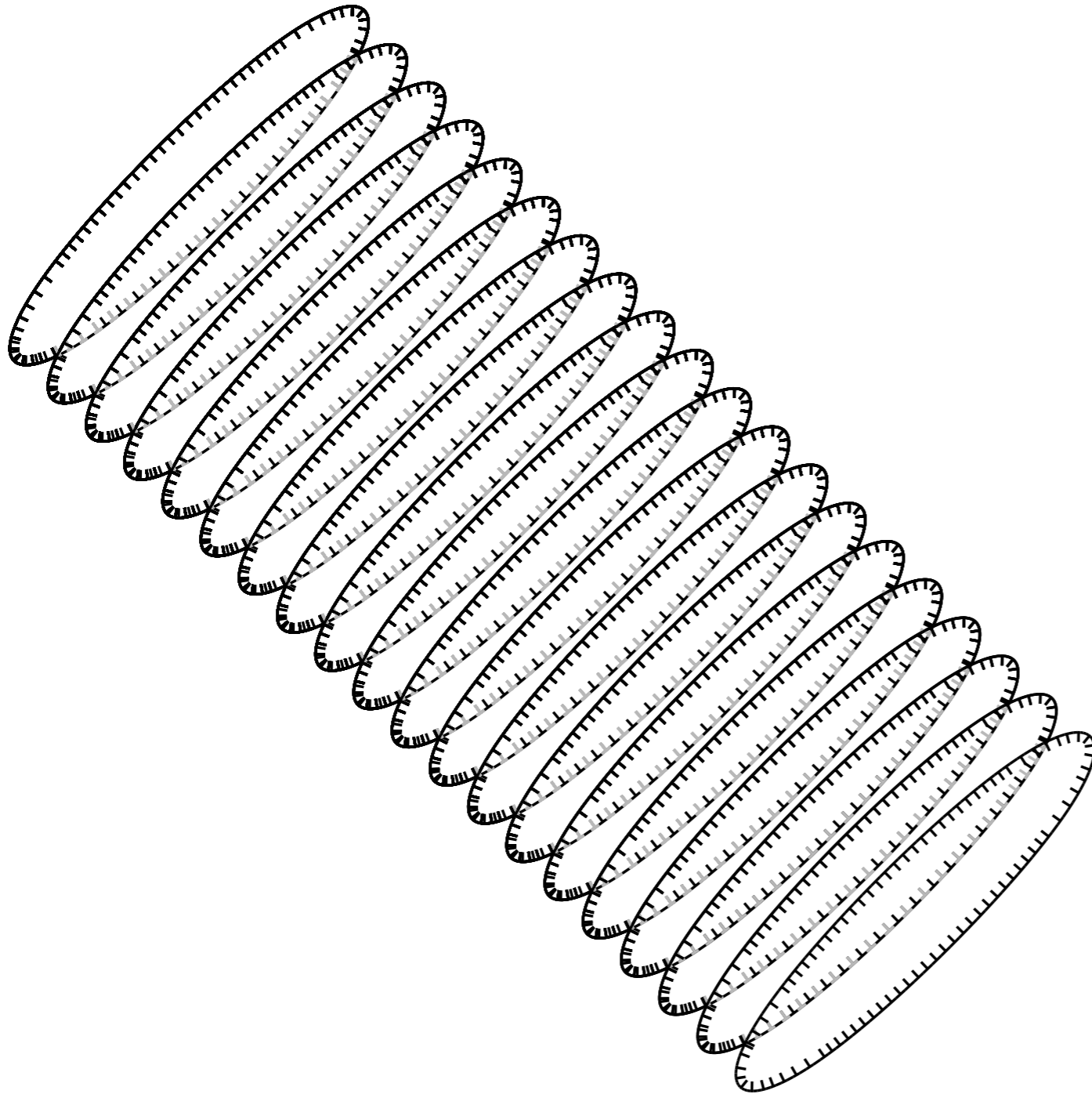
# Measuring Drawing Complexity

- Total number of crossings
- Maximum depth

# Experiments: Two Labeling Methods

- Randomized labeling
- Incremental labeling

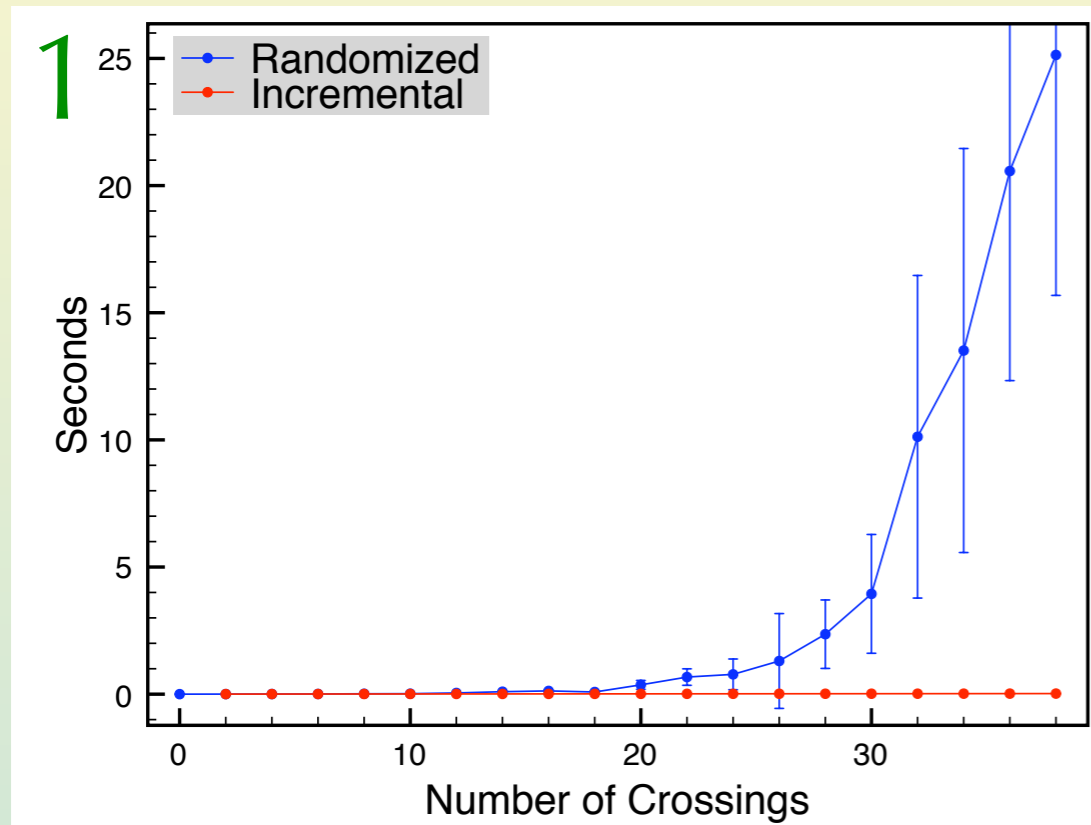
# Test 1



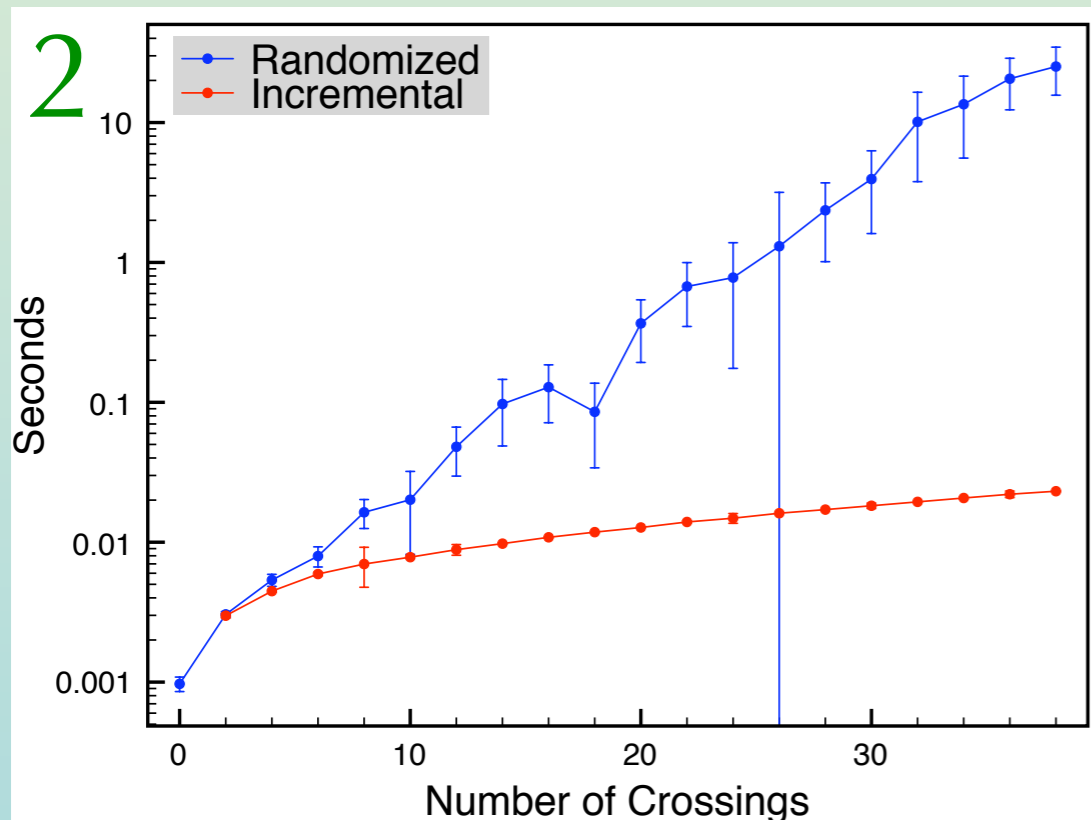
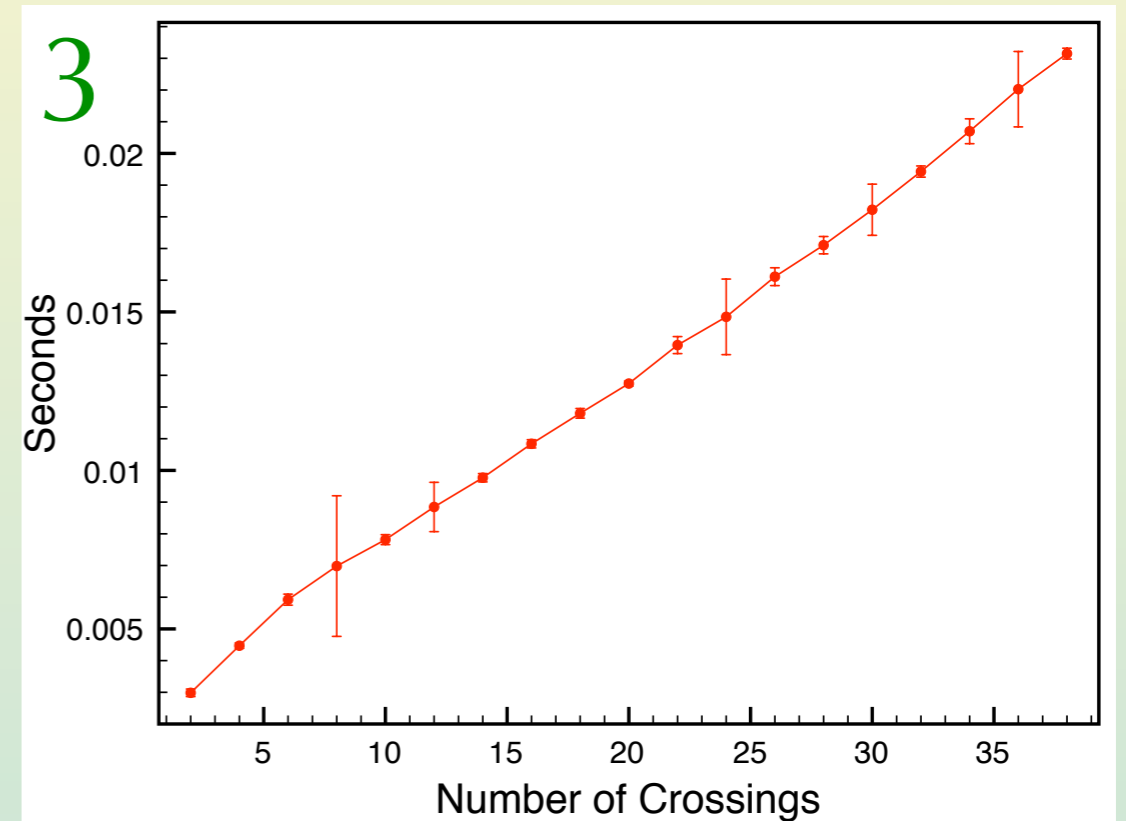
- **Number of crossings:**  
linear in the number  
of surfaces
- **Max depth:**  
constant

# Test 1: Labeling Time vs. # Crossings

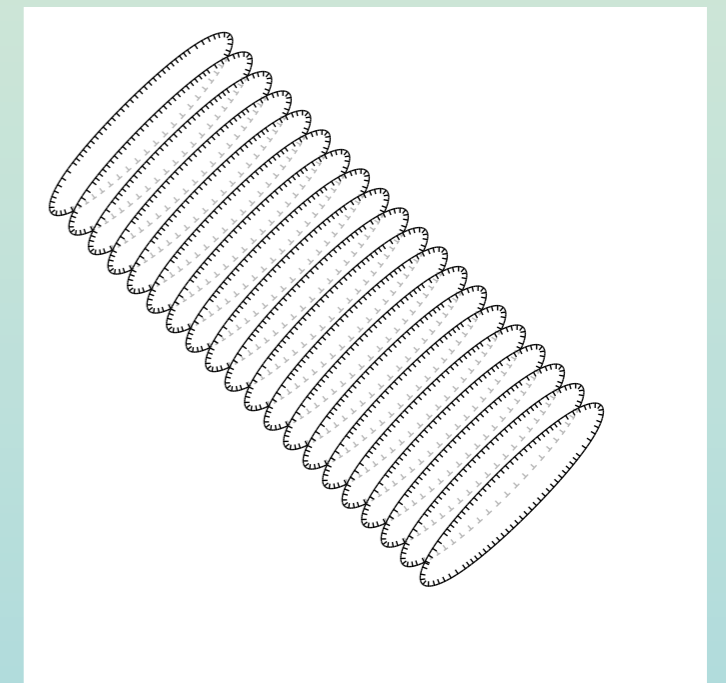
Running time vs. # Crossings



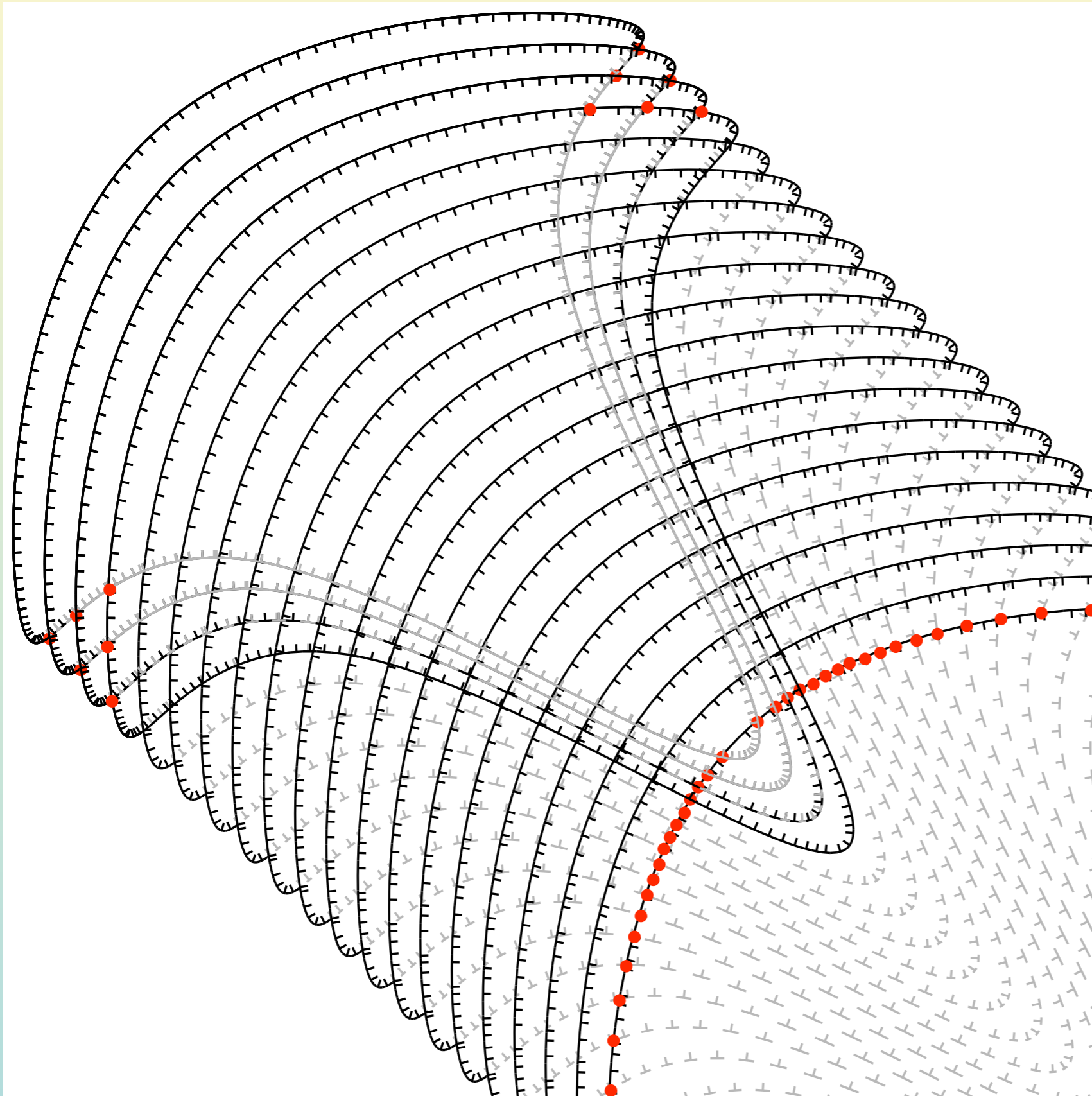
Running Time vs. # Crossings  
(Incremental Labeling)



Running time vs.  
# Crossings  
(log Y axis)



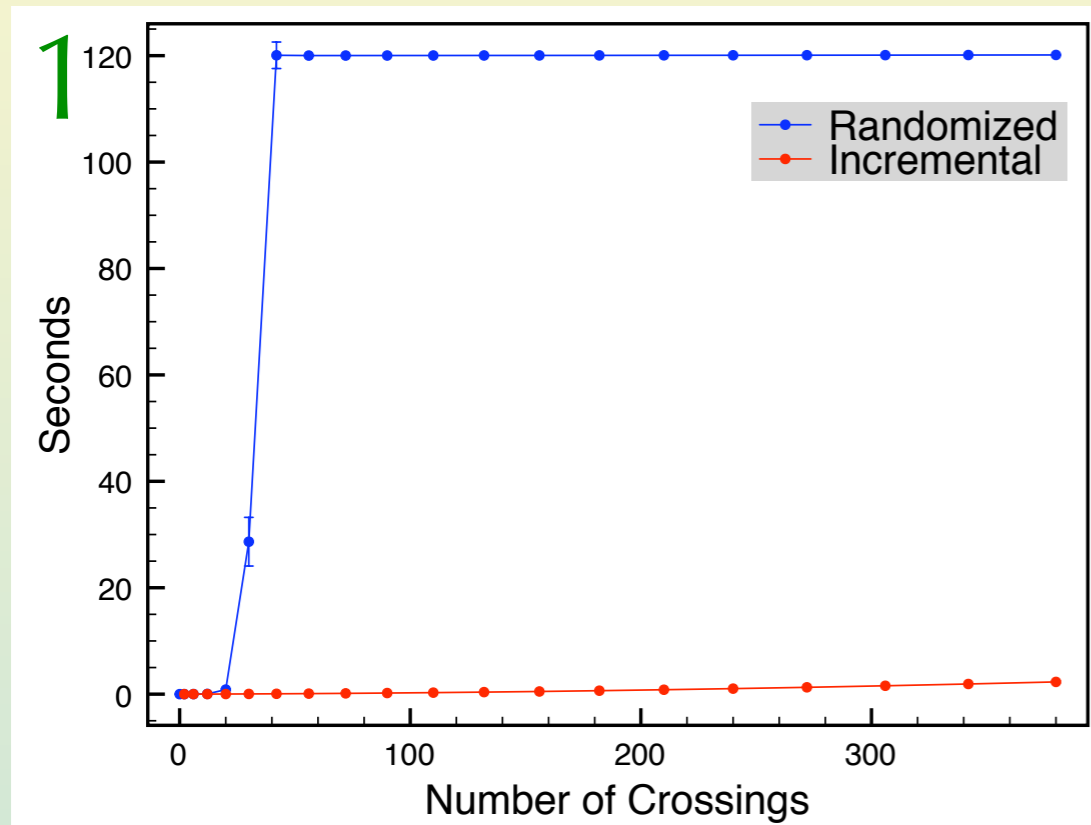
# Test 2



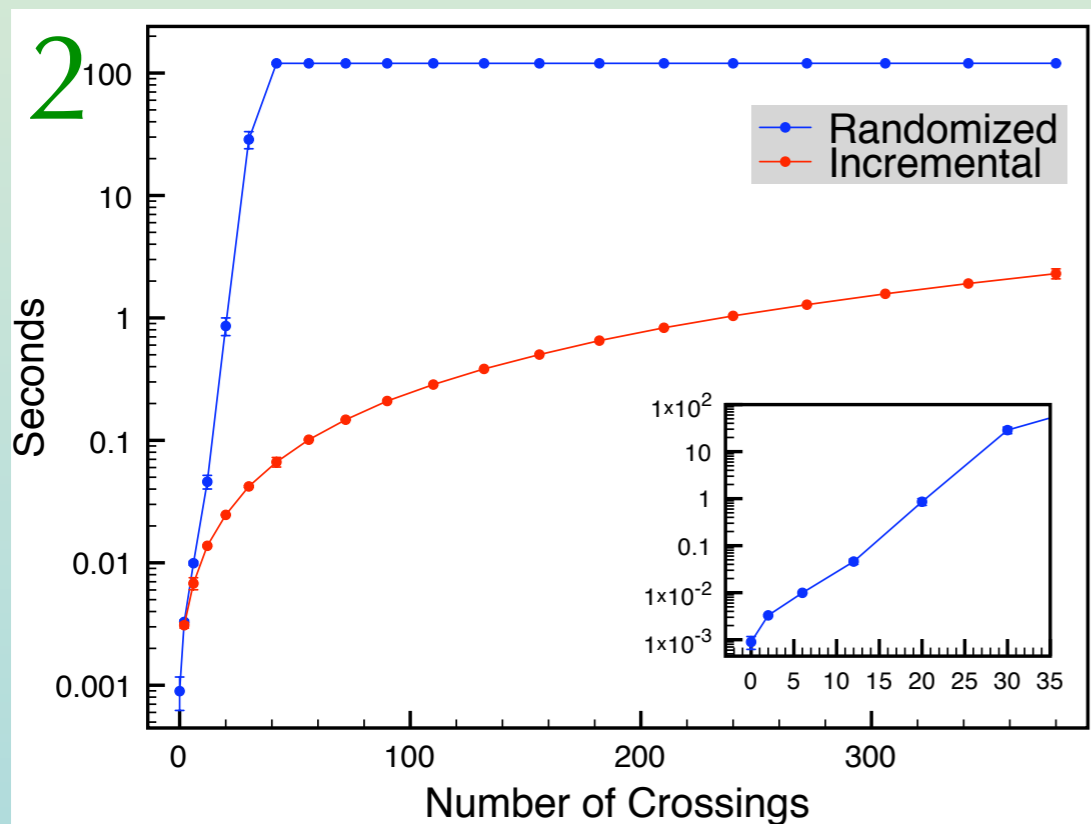
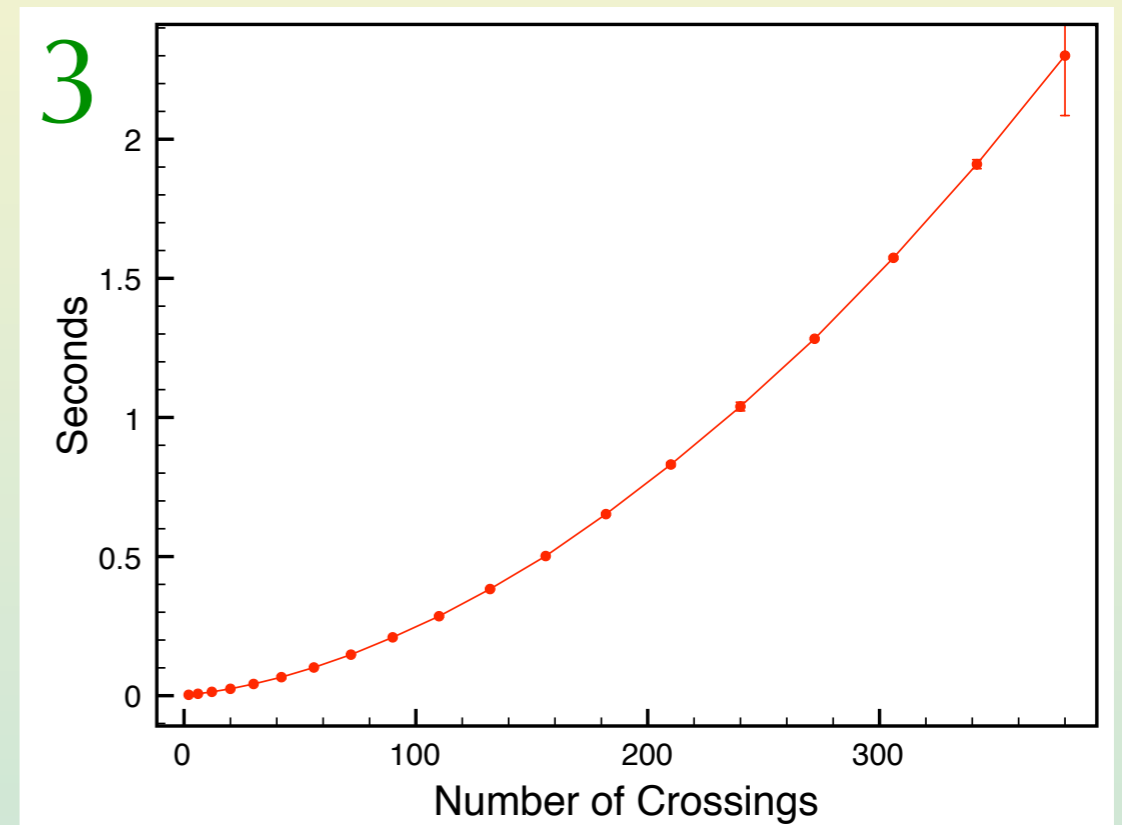
- **Number of crossings:**  
quadratic in the  
number of surfaces
- **Max depth:**  
linear in the number  
of surfaces

# Test 2: Labeling Time vs. # Crossings

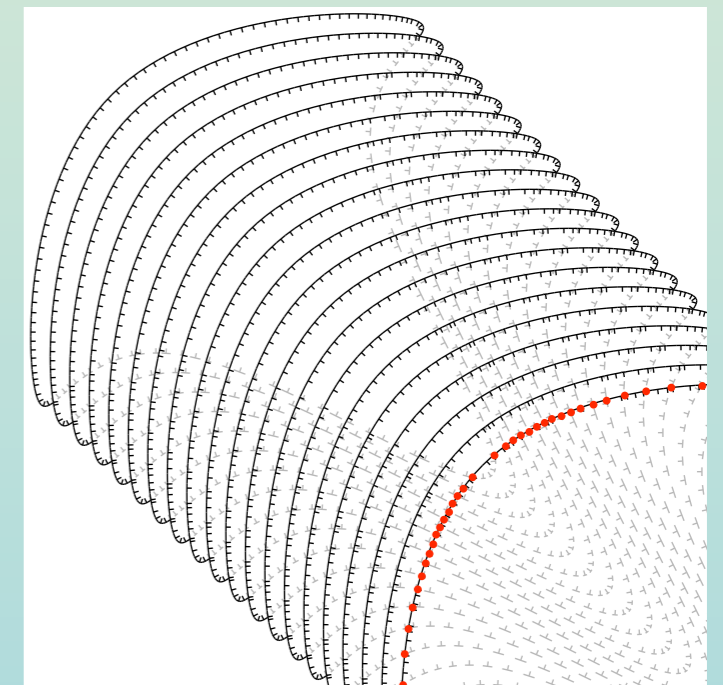
Running time vs. Number of Crossings



Running Time vs. Number of Crossings  
(Incremental Labeling)



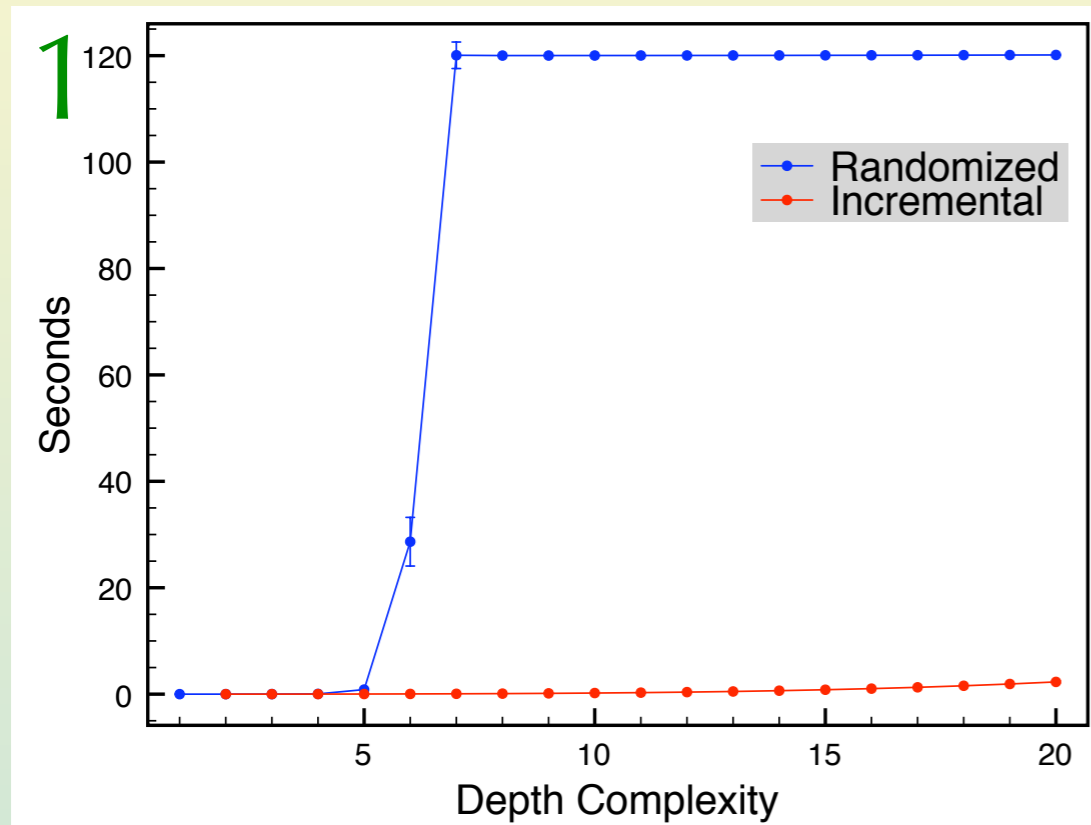
Running time vs.  
Number of Crossings  
(log Y axis)



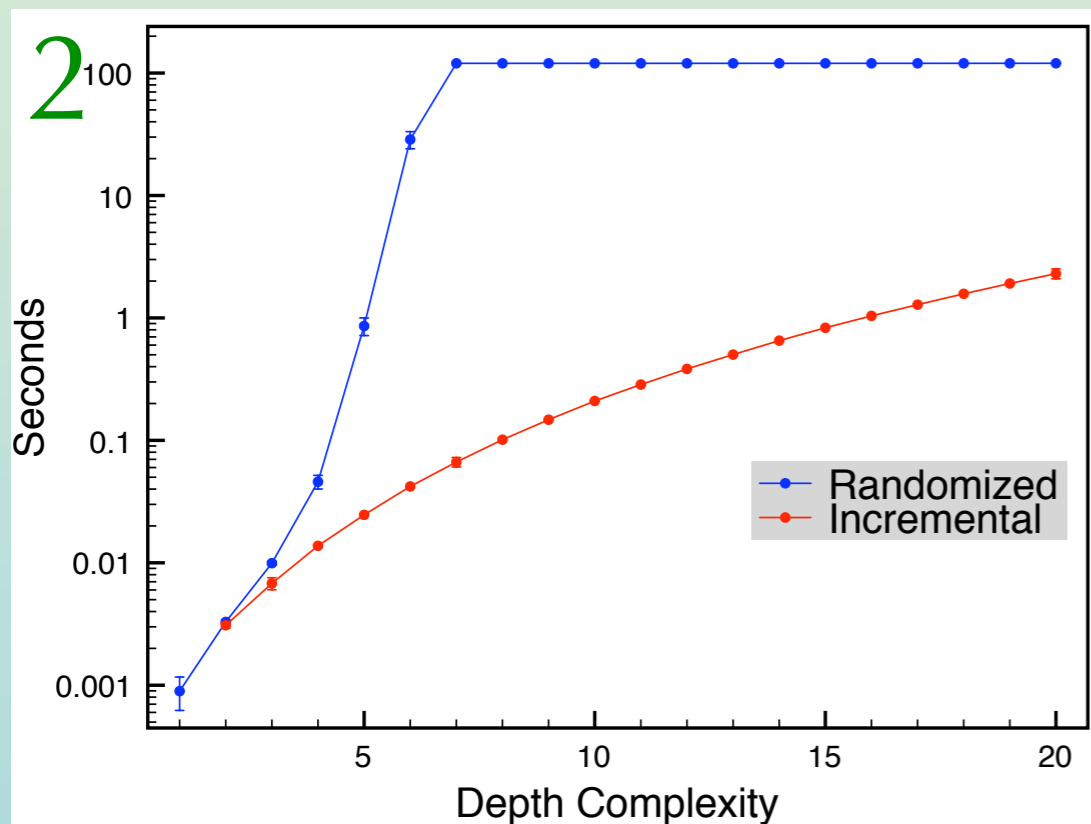
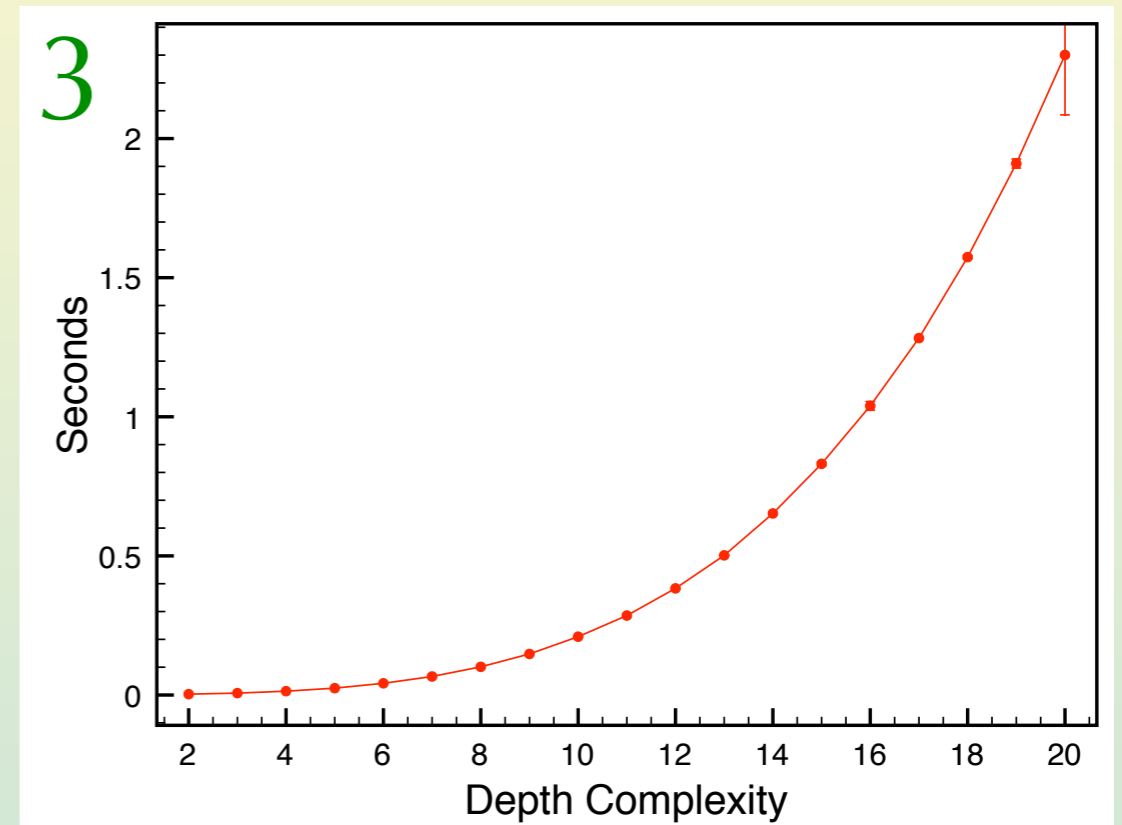


# Test 2: Labeling Time vs. Max Depth

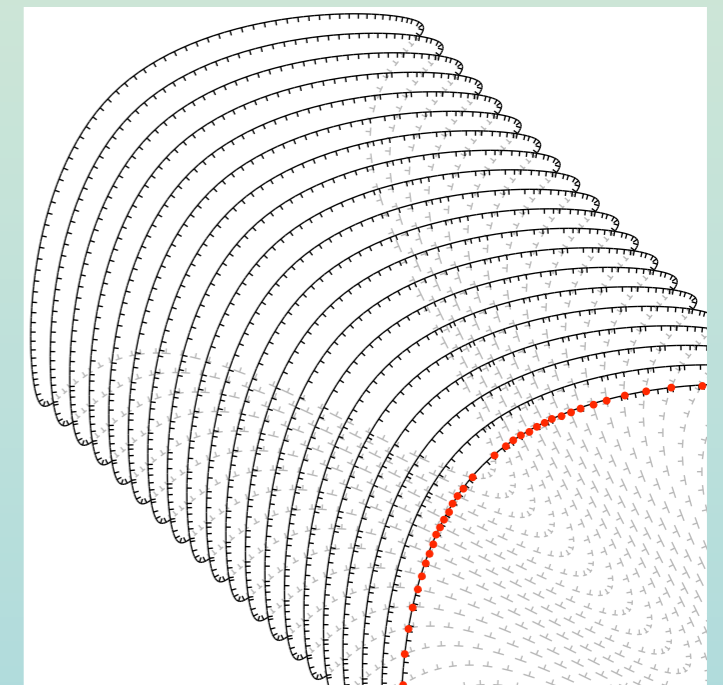
Running time vs. Max Depth



Running time vs. Max Depth

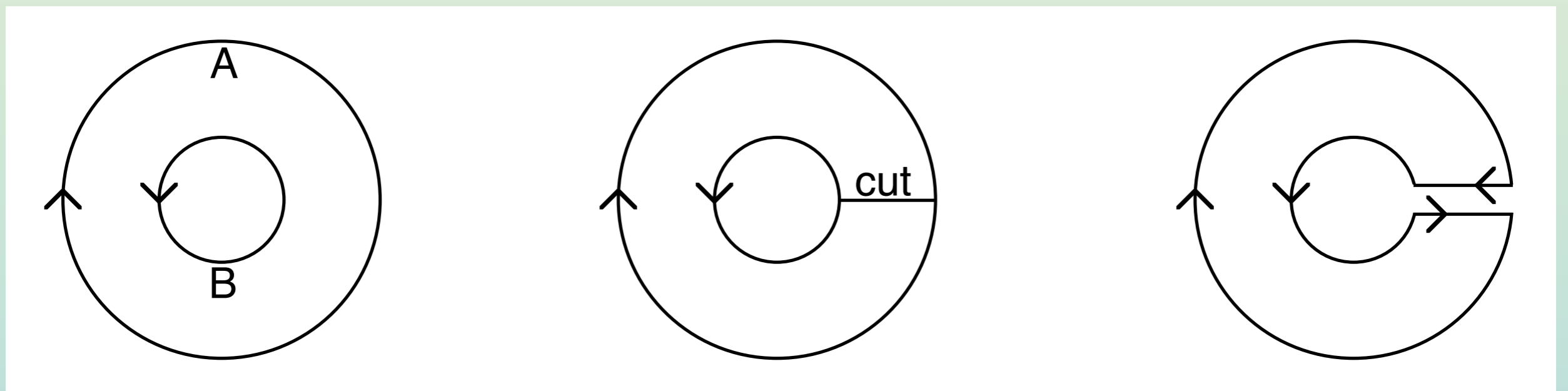


Running time vs. Max Depth (log Y axis)



# Boundary Grouping with Cuts

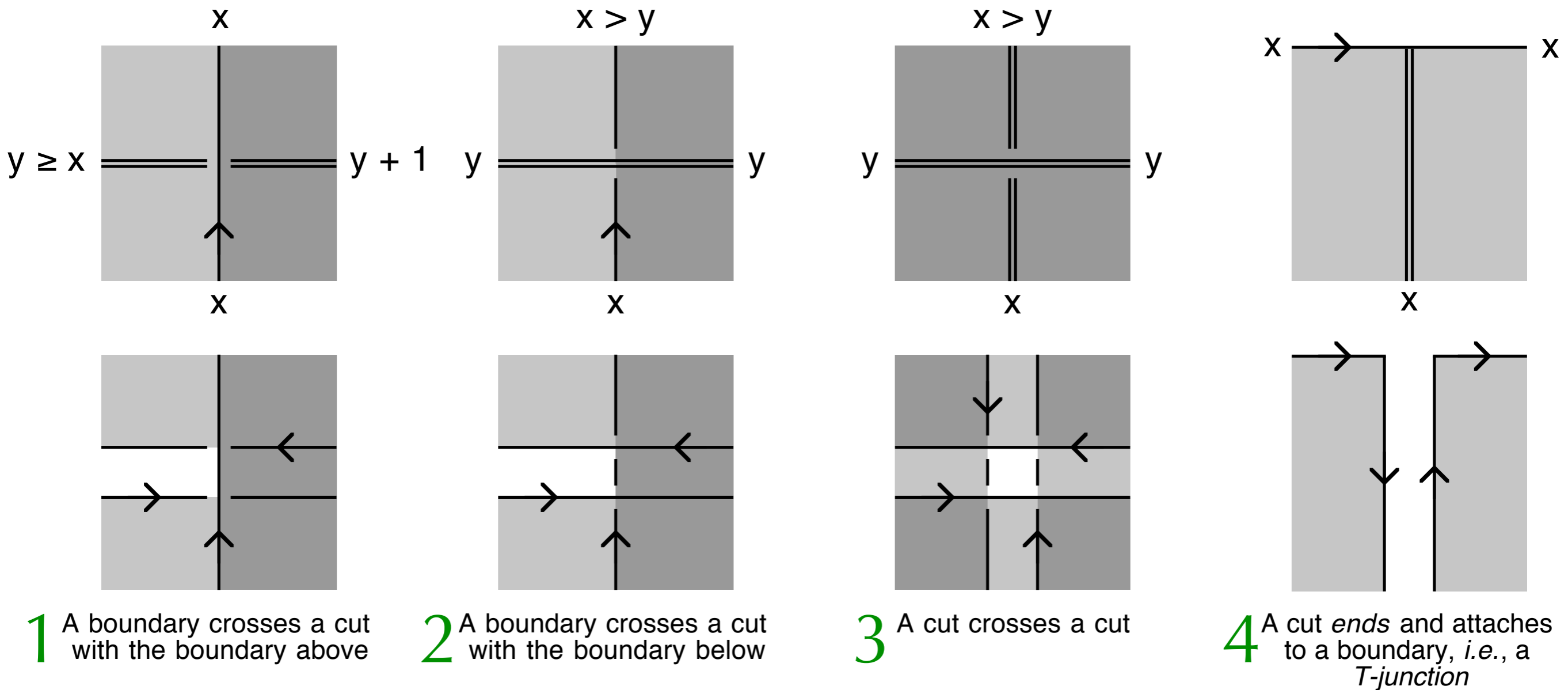
- Some surfaces have multiple boundaries
- This can cause problems
- A **cut** between two different boundaries reduces the number of boundaries by one



Cuts are a geometric device. Needn't be horizontal or straight.

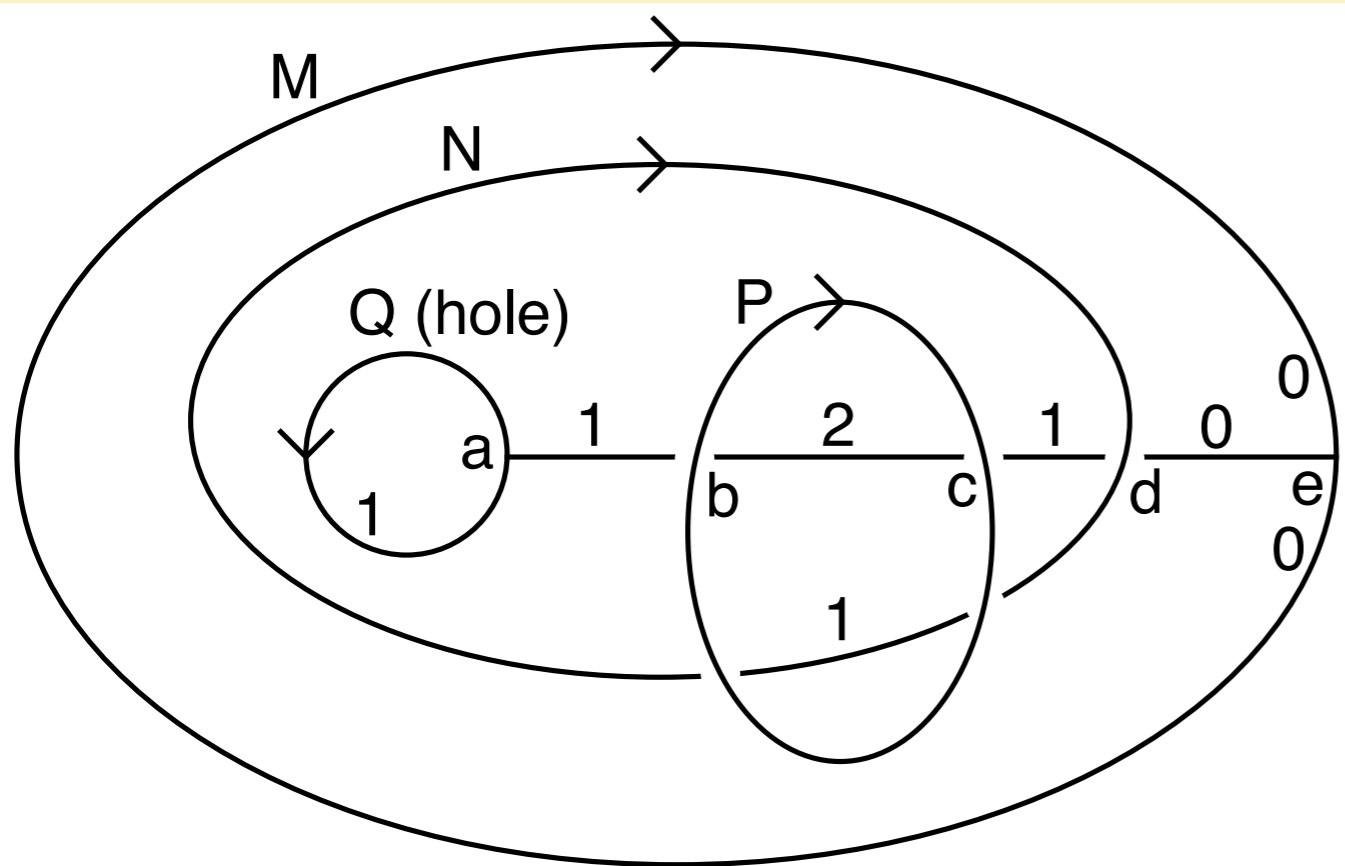
# Cut Labeling Schemes

Using cuts requires four new labeling schemes

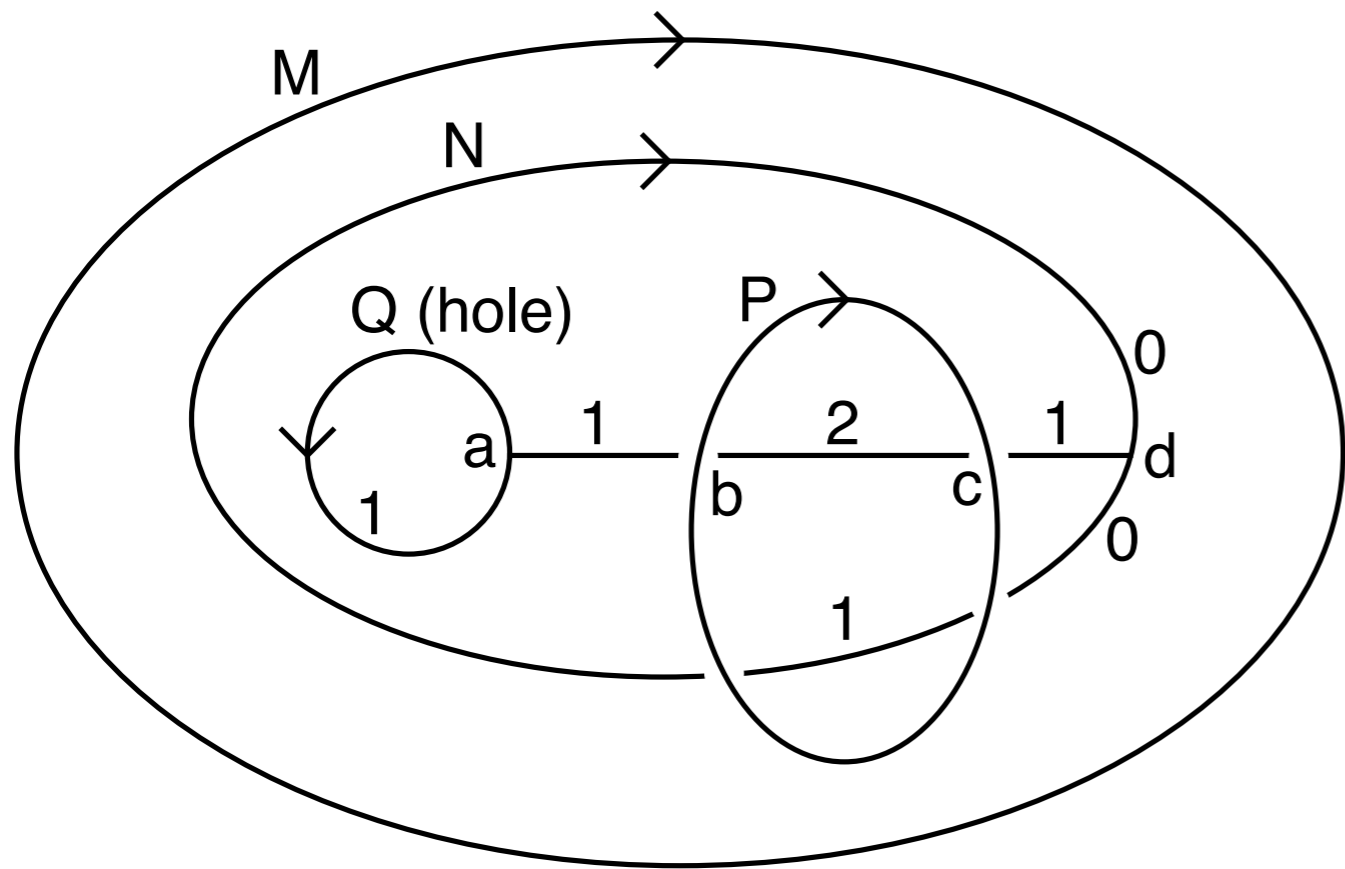


Cuts denoted with a double line (top row) and a gap (bottom row)

# Finding Legal Cuts



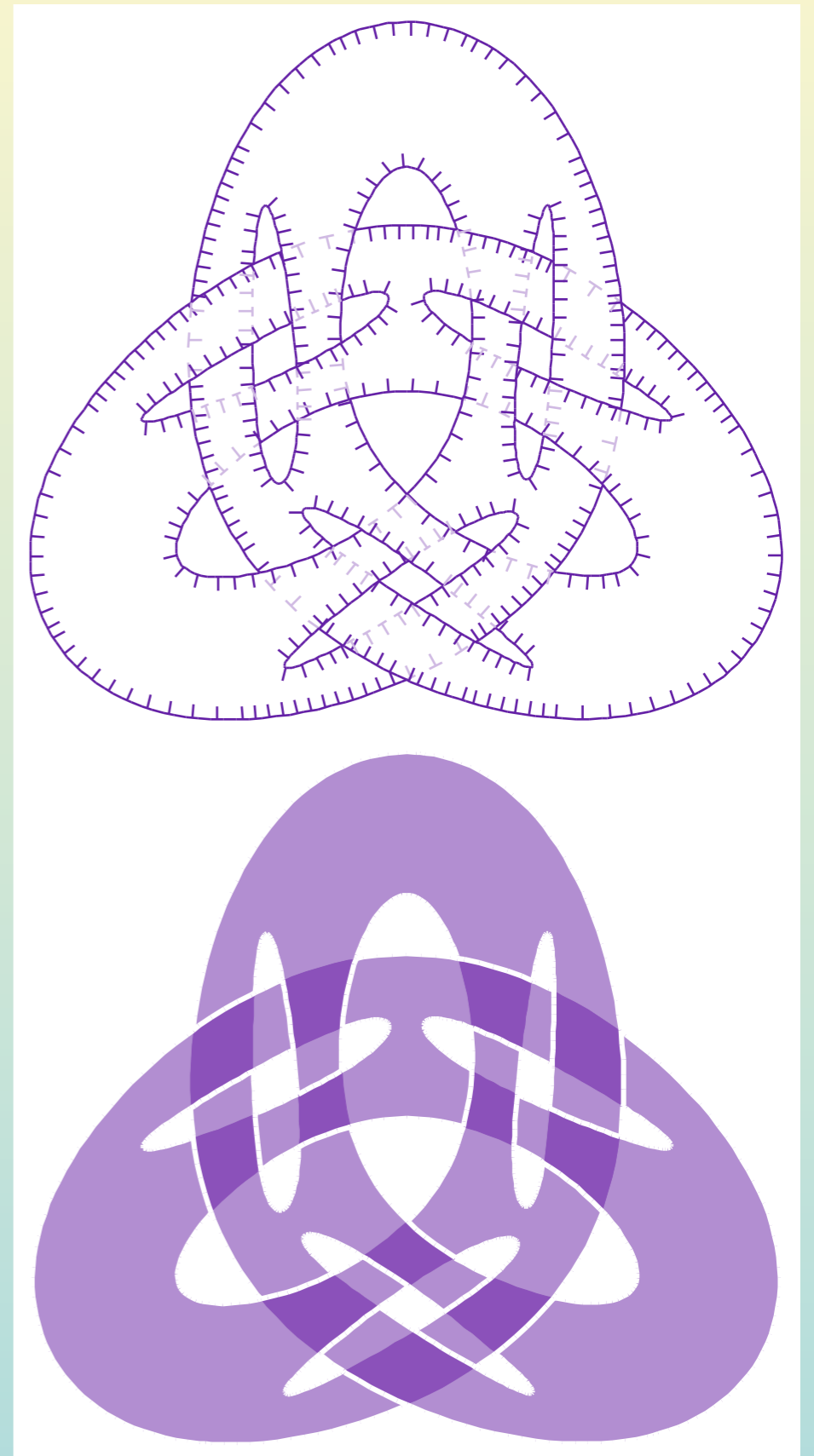
A successful cut:  
Last crossing (*e*) is  
legal.



An unsuccessful cut:  
Last crossing (*d*) is  
illegal.

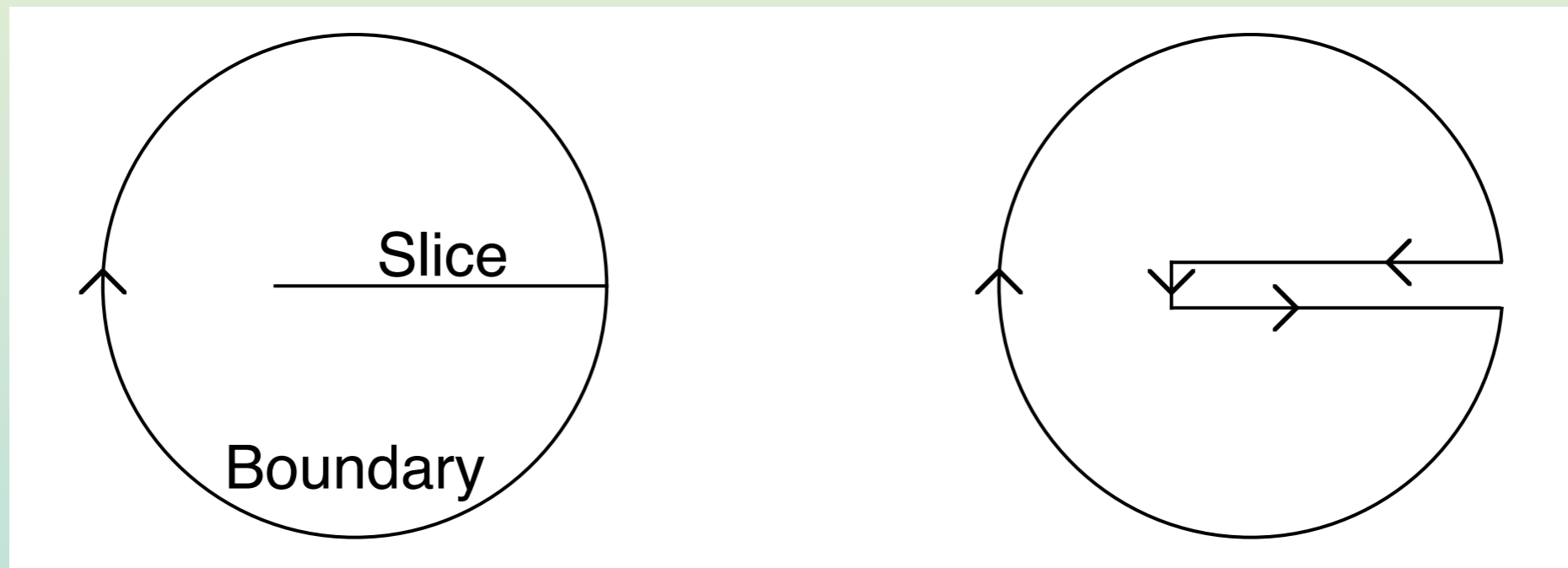
# Rendering

- Conversion of a labeled knot-diagram to an image with solid fills
- Requires full depth ordering of all surfaces covering each region
- *Druid* uses the ***episcotister model***  
(Metelli '74)



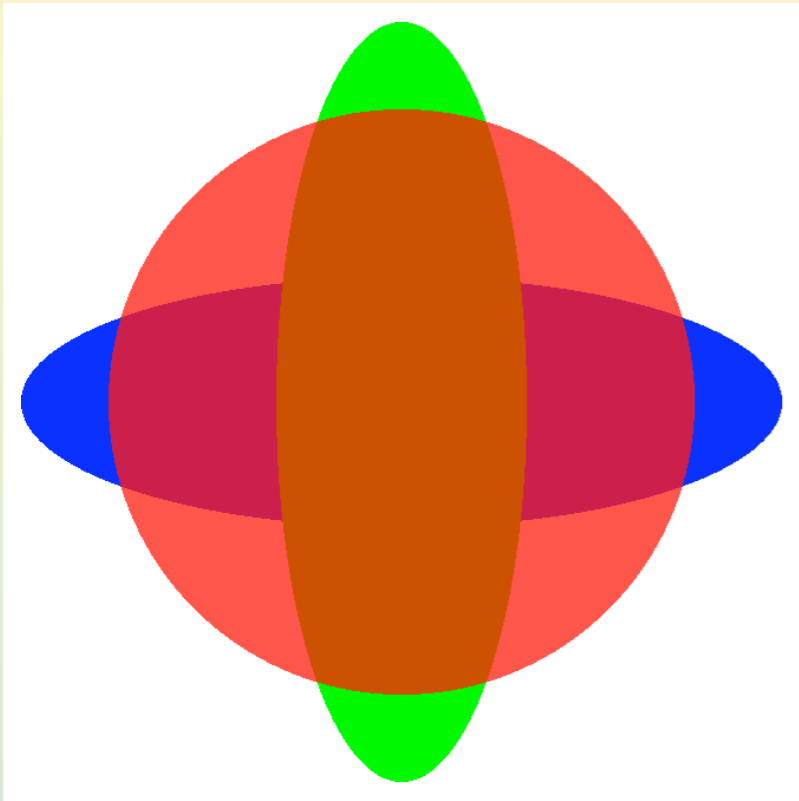
# Slice

- A ***slice*** connects a location on a boundary to a point within the bounded surface
- Similar to a cut

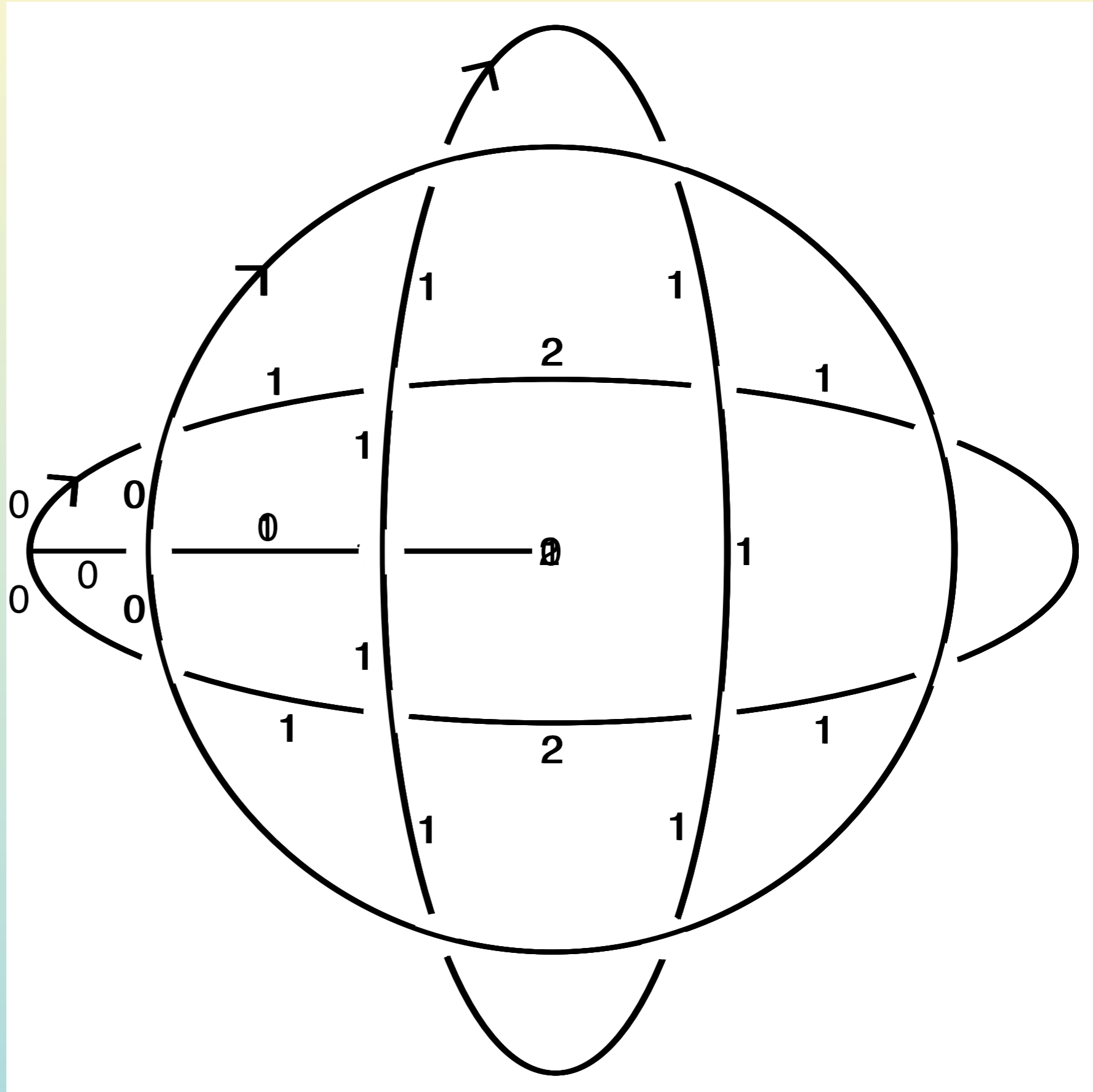


Slices are a geometric device. Needn't be horizontal or straight.

# Using Slices to Find Region Coverings

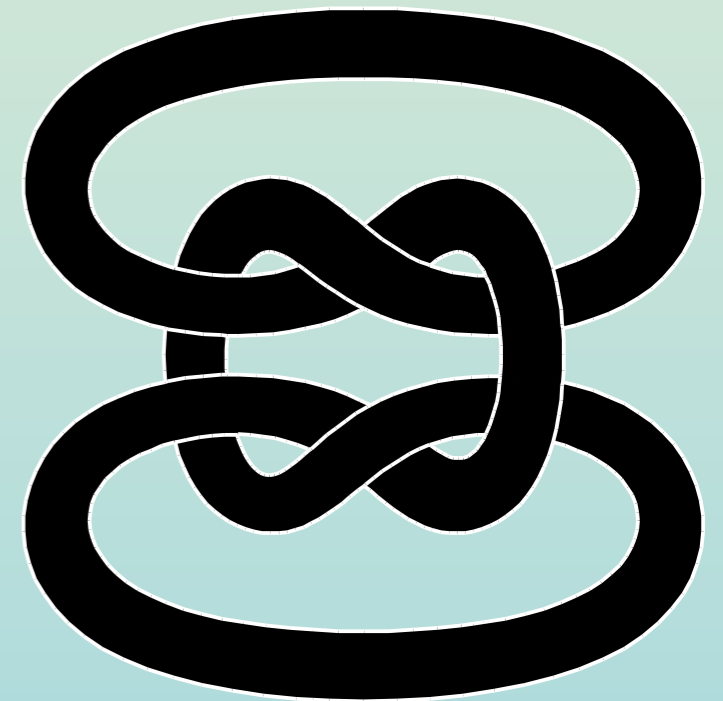
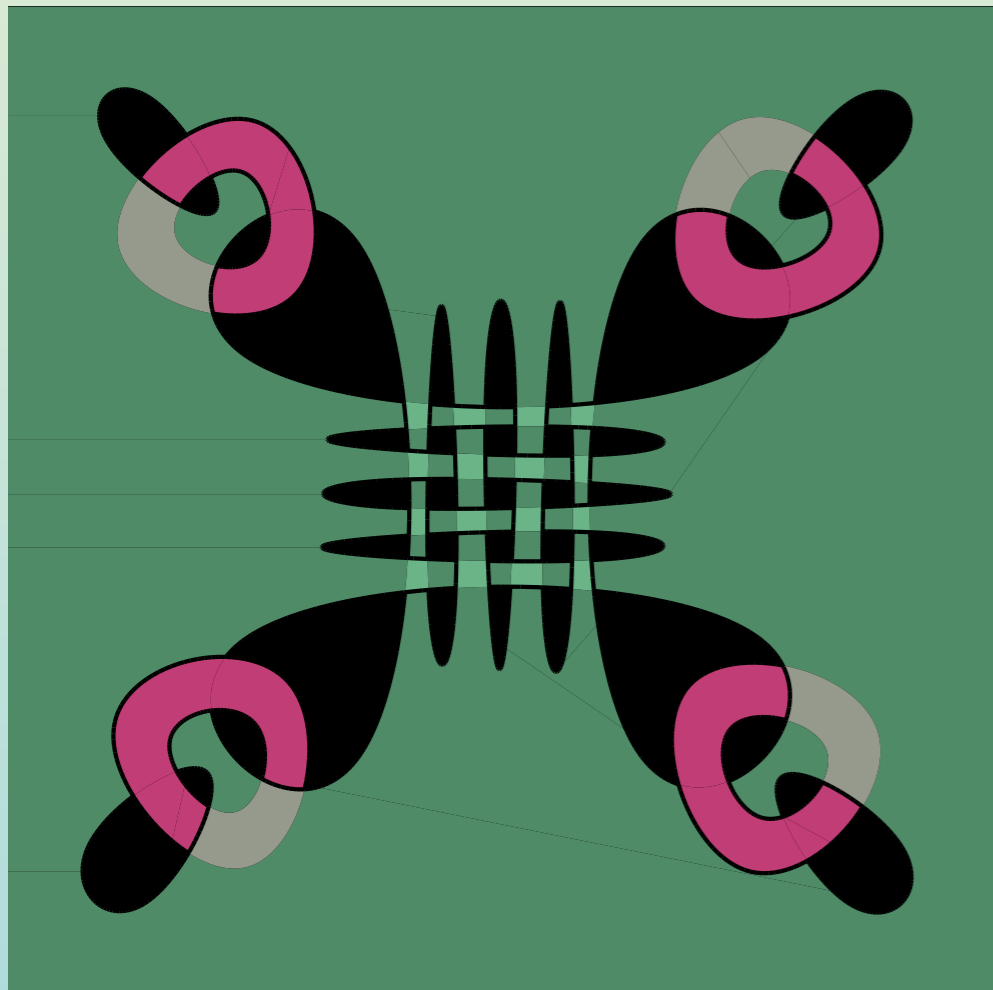
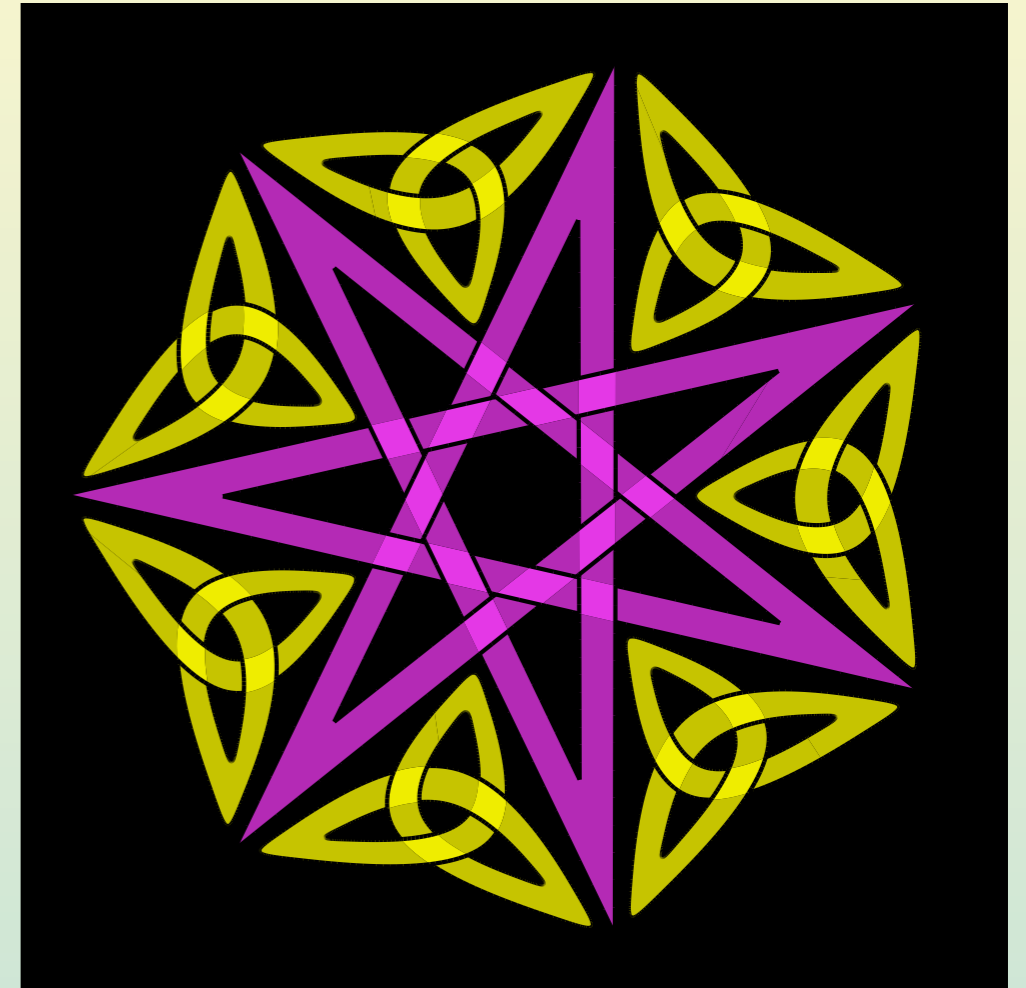
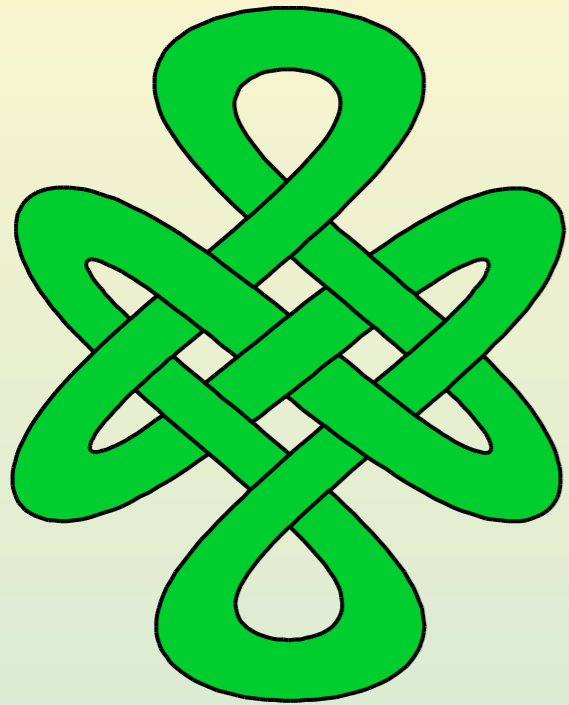


**Red** is above  
**green**, which  
is above **blue**





# *Druid* Examples



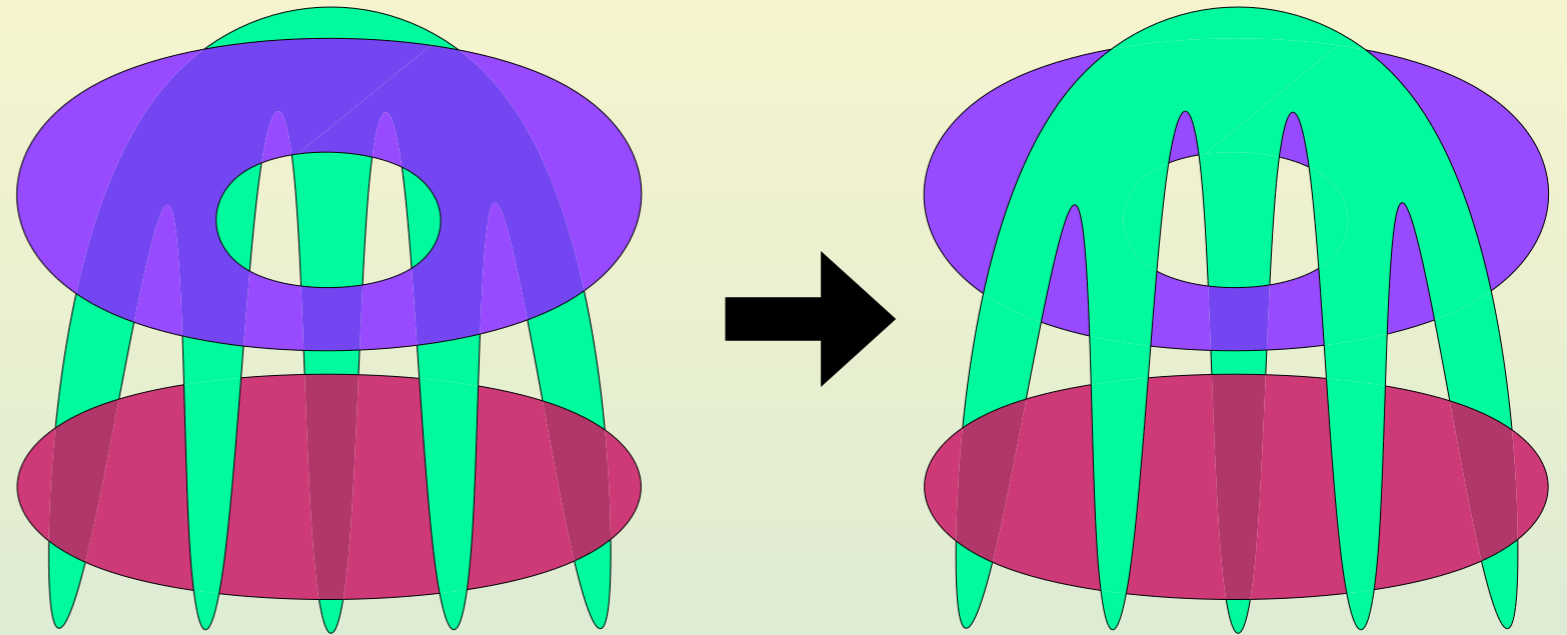
# A Problem with the Search

$C$

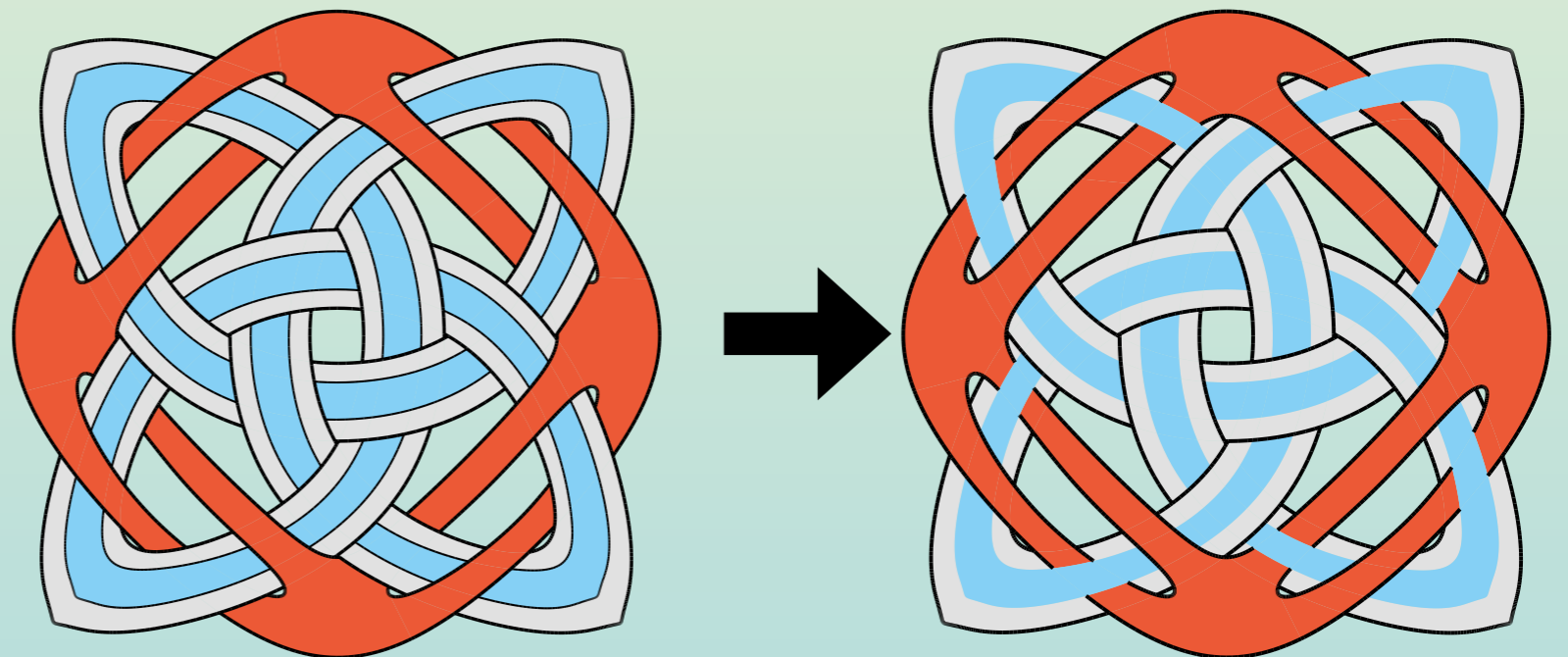
- Search space size:  $2^C$  for  $C$  crossings
- A drawing can have hundreds of crossings.
- The search takes too long for complex drawings.
- Thus, *Druid* as described in (Wiley and Williams '06a) was limited.

# A Problem with the Search (contd.)

*Druid* fails to label this flip in under 120 seconds in 50% of tests



*Druid* takes 35 seconds on average to perform one of these flips (and fails in 2% of tests)

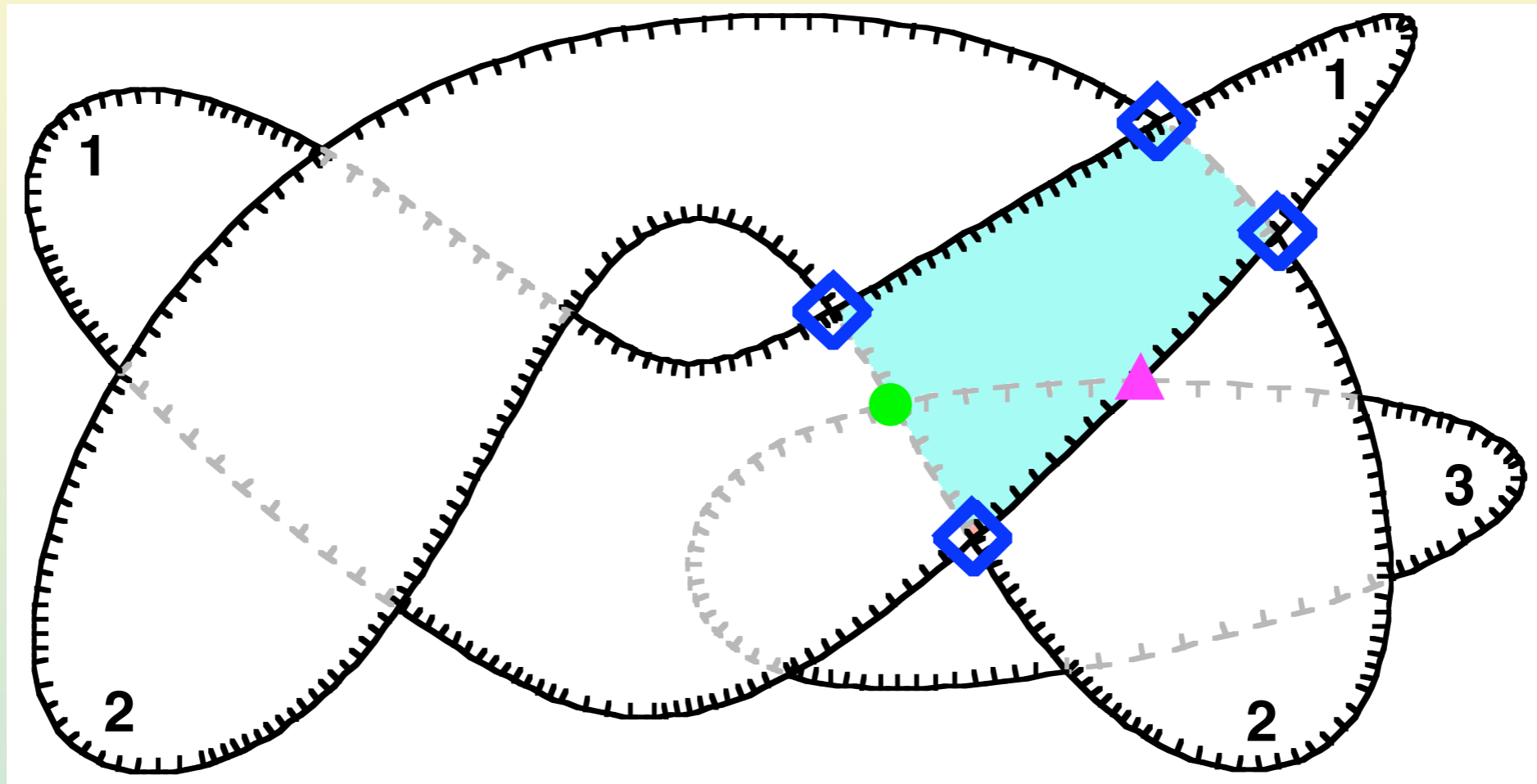


# Crossing-State Equivalence Class Rule

- Discovered a property of 2½D scenes, the *crossing-state equivalence class (CSEC) rule*
- Use this property to improve performance

# Area of Overlap

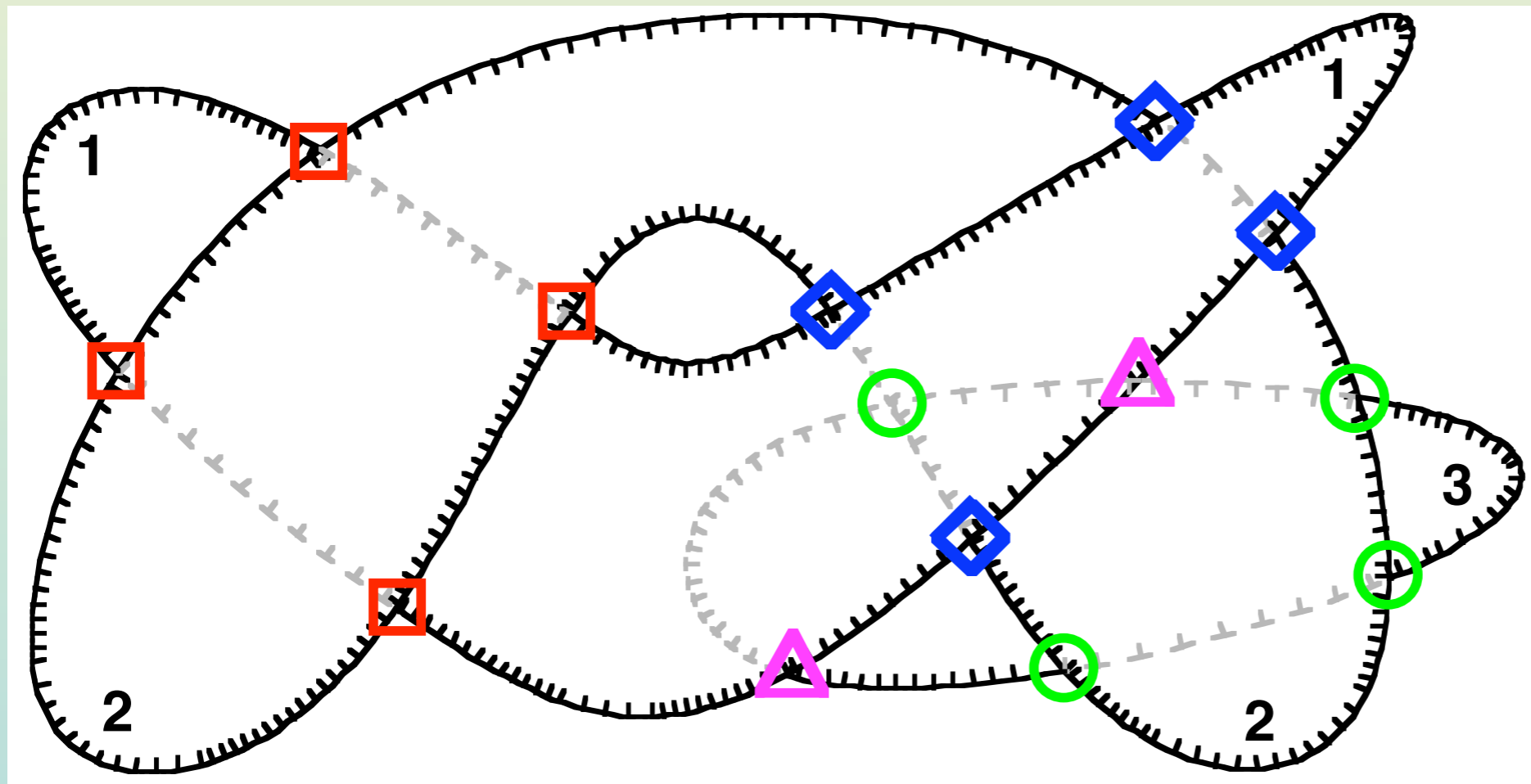
Numbers  
label unique  
surfaces



- **Area of overlap:** The maximum contiguous area where two surfaces overlap, e.g., the shaded area for surfaces 1 and 2
- **Corner:** A crossing where a traversal of an *area of overlap's* border switches boundaries, e.g., the **blue diamonds** for the shaded area

# Crossing-State Equivalence Class (CSEC)

The corners of an area of overlap



Unique shapes/colors indicate CSECs

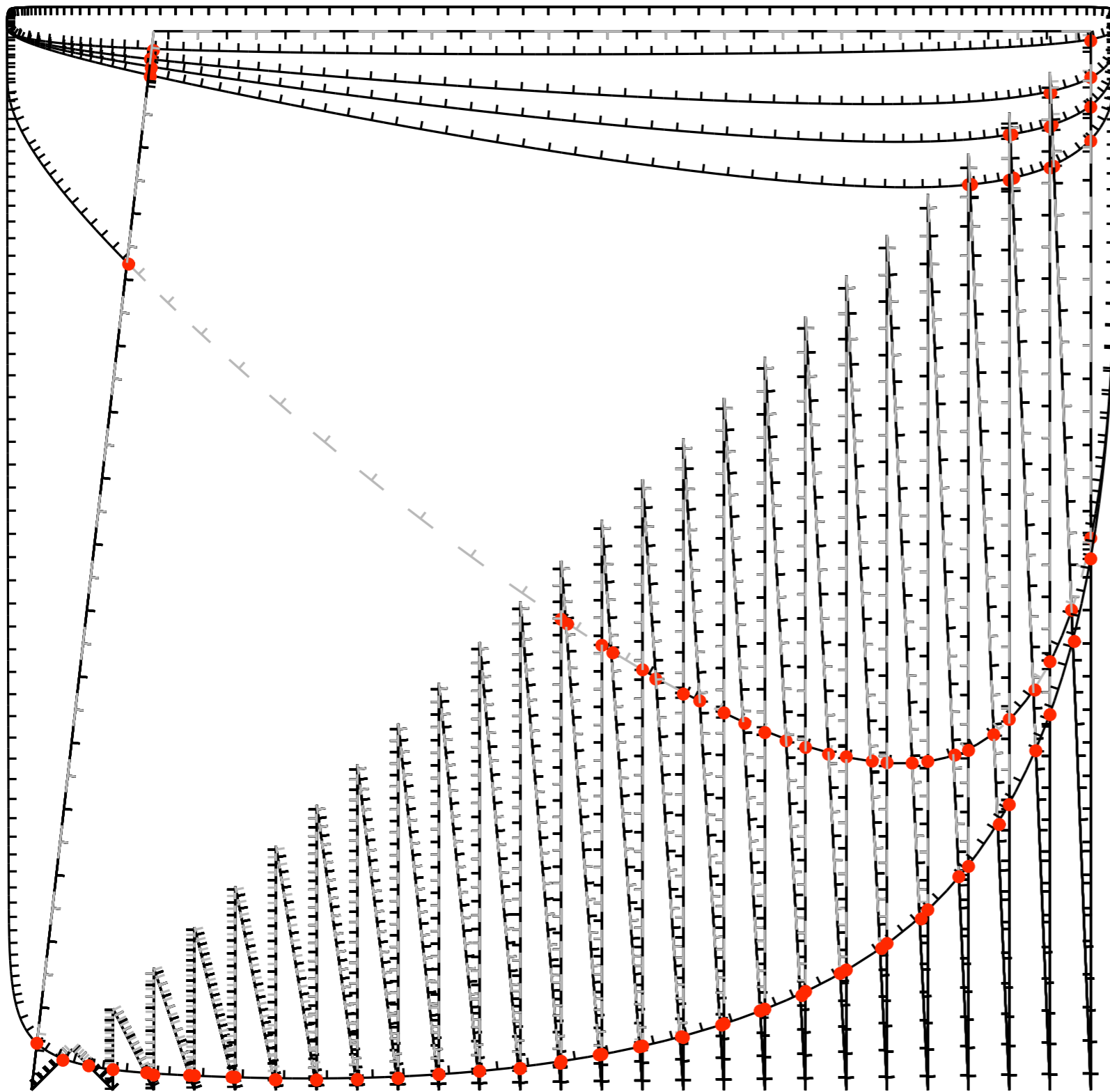


# Finding CSECs on Labeled Figures

- Intend to use CSECs to improve performance
- But *Druid* must ***find*** the CSECs before they can be used
- How long does this take? Does it cancel the benefit of using CSECs in the first place?



# Finding CSECs on Labeled Figures

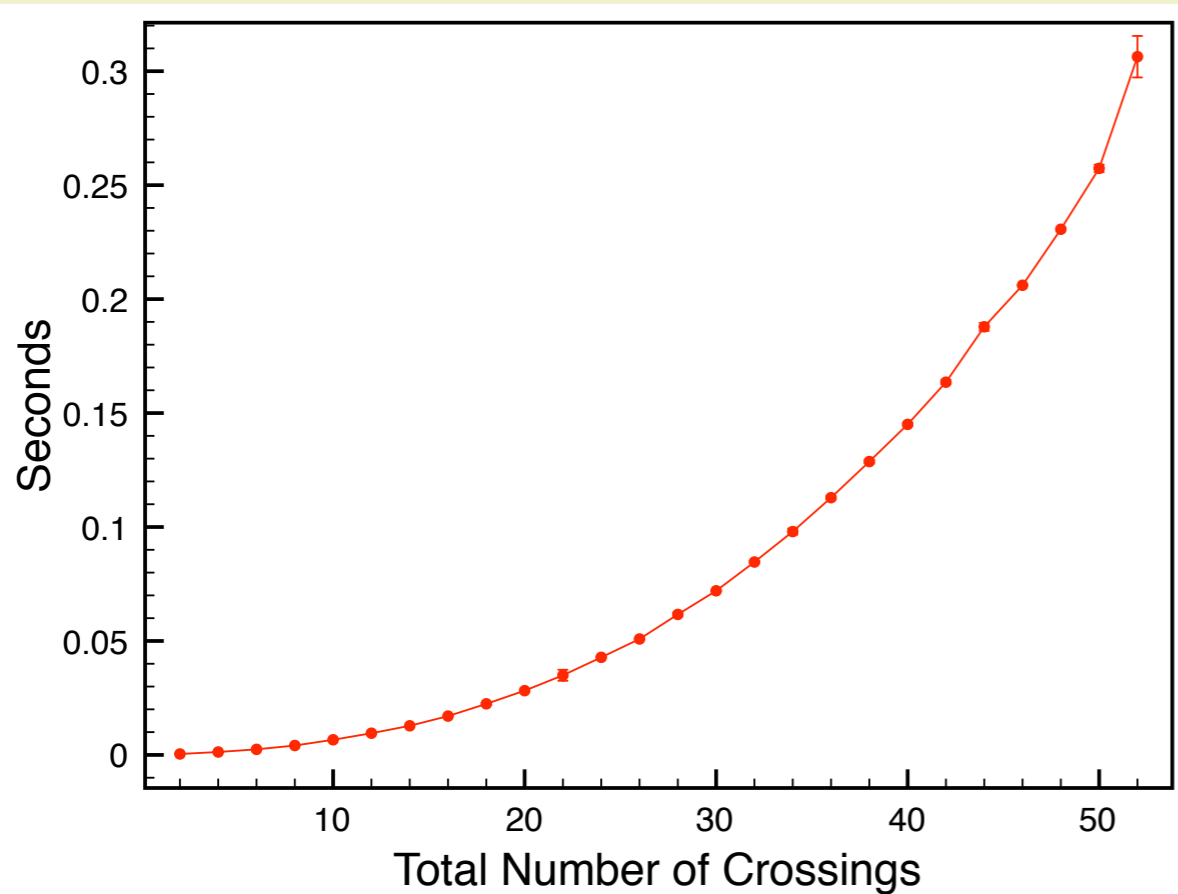


**Experiment:** Across a spectrum of CSEC sizes, measure the time required to find all CSECs.

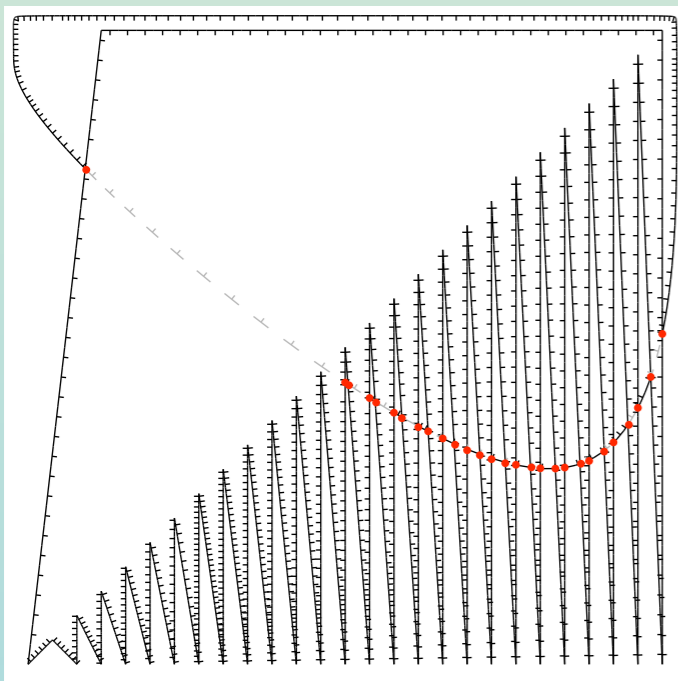
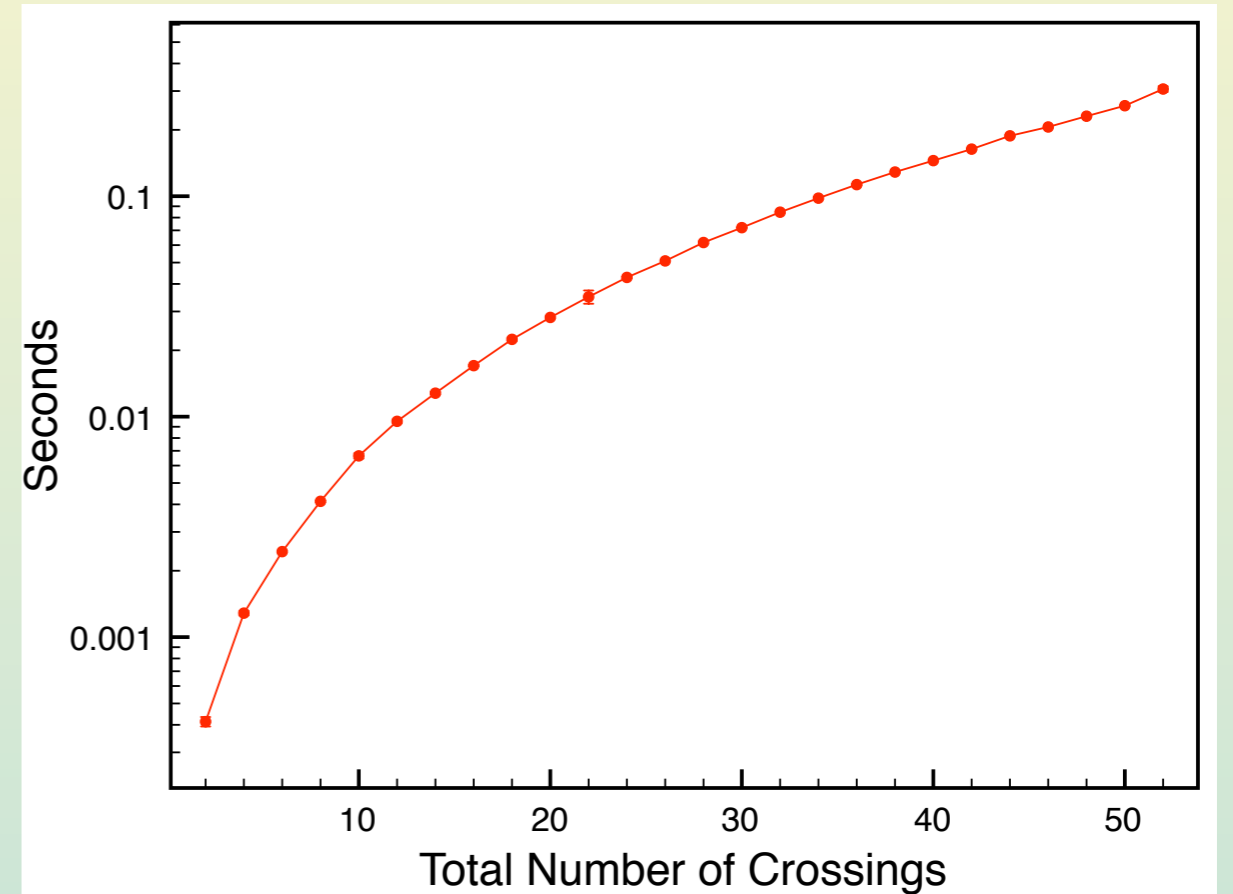
In this experiment there is only one CSEC.

# Finding CSECs on Labeled Figures

Running Time vs. # Crossings



Running Time vs. # Crossings  
(log Y axis)

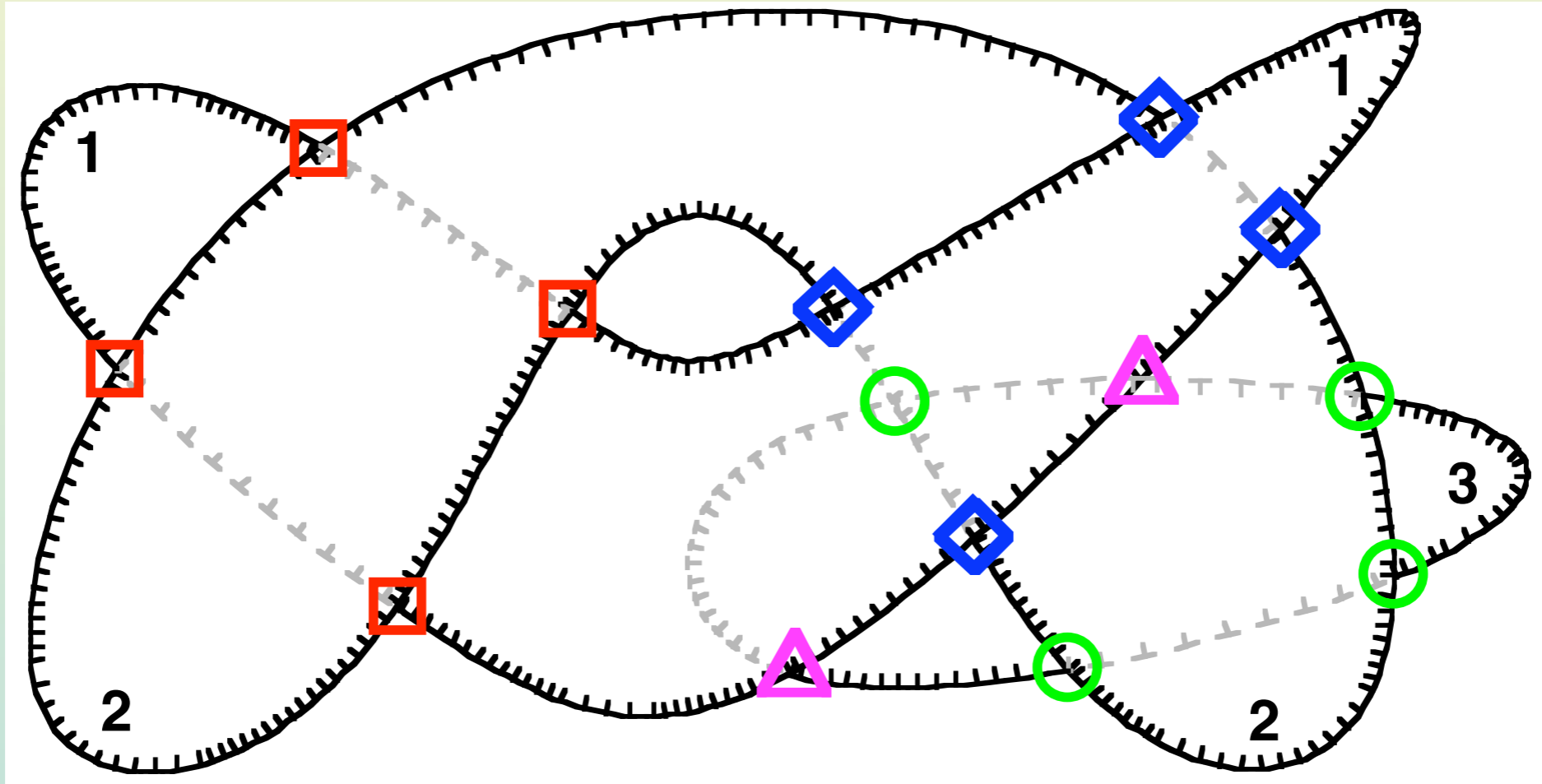


Running time to find CSECs for these figures is polynomial in the number of crossings.

**Note:** The actual time is very low (.3 secs for 52 crossings)

# Crossing-State Equivalence Class Rule

*All members of a crossing-state equivalence class must be in the same state.*



e.g., for surfaces 2 and 3 all corners of the **green circle** CSEC must be in the same state, *i.e.*, either 2 is above 3 or  $vs/va$ .

# Two Relabeling Methods

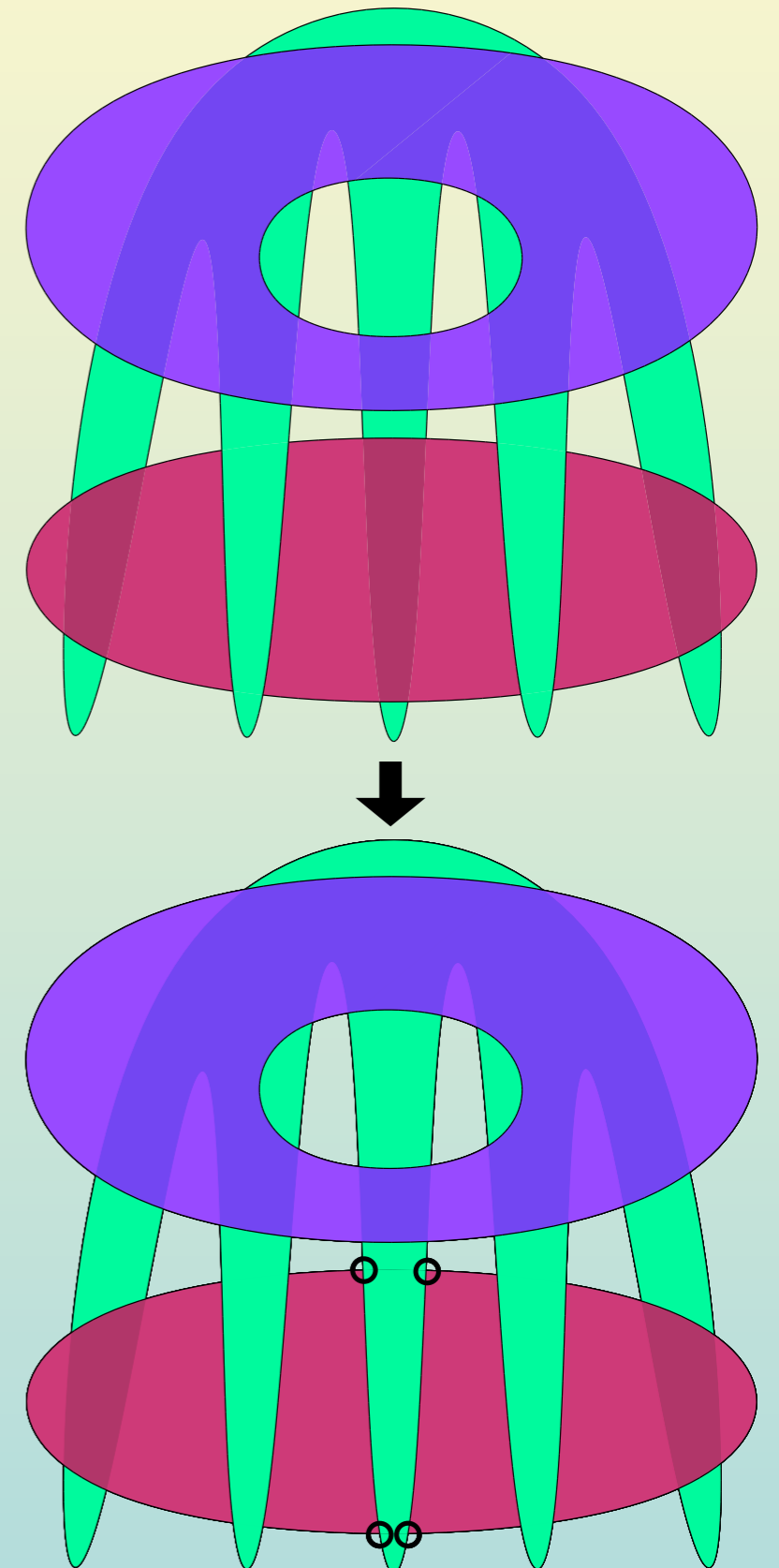
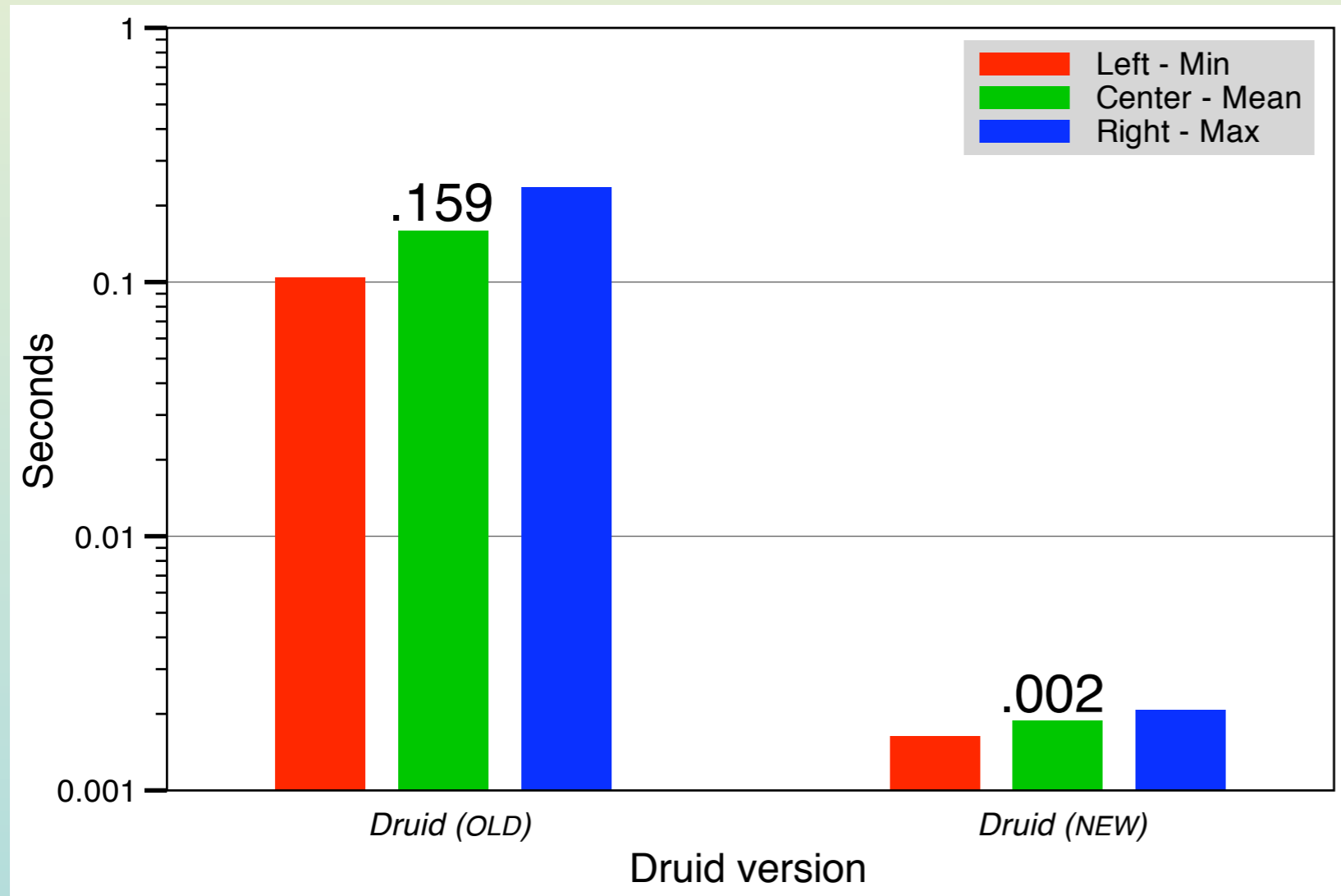
1. *Druid (OLD)*: Performs a tree search (Wiley and Williams '06a).
2. *Druid (NEW)*: Maintains the CSECs without a search. Deduces resulting segment depth changes directly (Wiley and Williams '06b).

Wiley, K. B., and L. R. Williams, 2006. Representation of Interwoven Surfaces in 2 1/2 D Drawing. *Proc. of CHI*, Conference on Human Factors in Computing Systems, Montreal, Canada, 2006.

Wiley, K. B., and L. R. Williams. Use of Crossing-State Equivalence Classes for Rapid Relabeling of Knot-Diagrams Representing 2 1/2 D Scenes. Tech Report, UNM, Dept of Computer Science, TR-CS-2006-08, 2006.

# Results: A Small CSEC Flip

- Size 4, indicated with circles
- Running times on 1.6GHz G5 PowerMac
- *Druid (NEW)* performs 85 times faster than *Druid (OLD)*

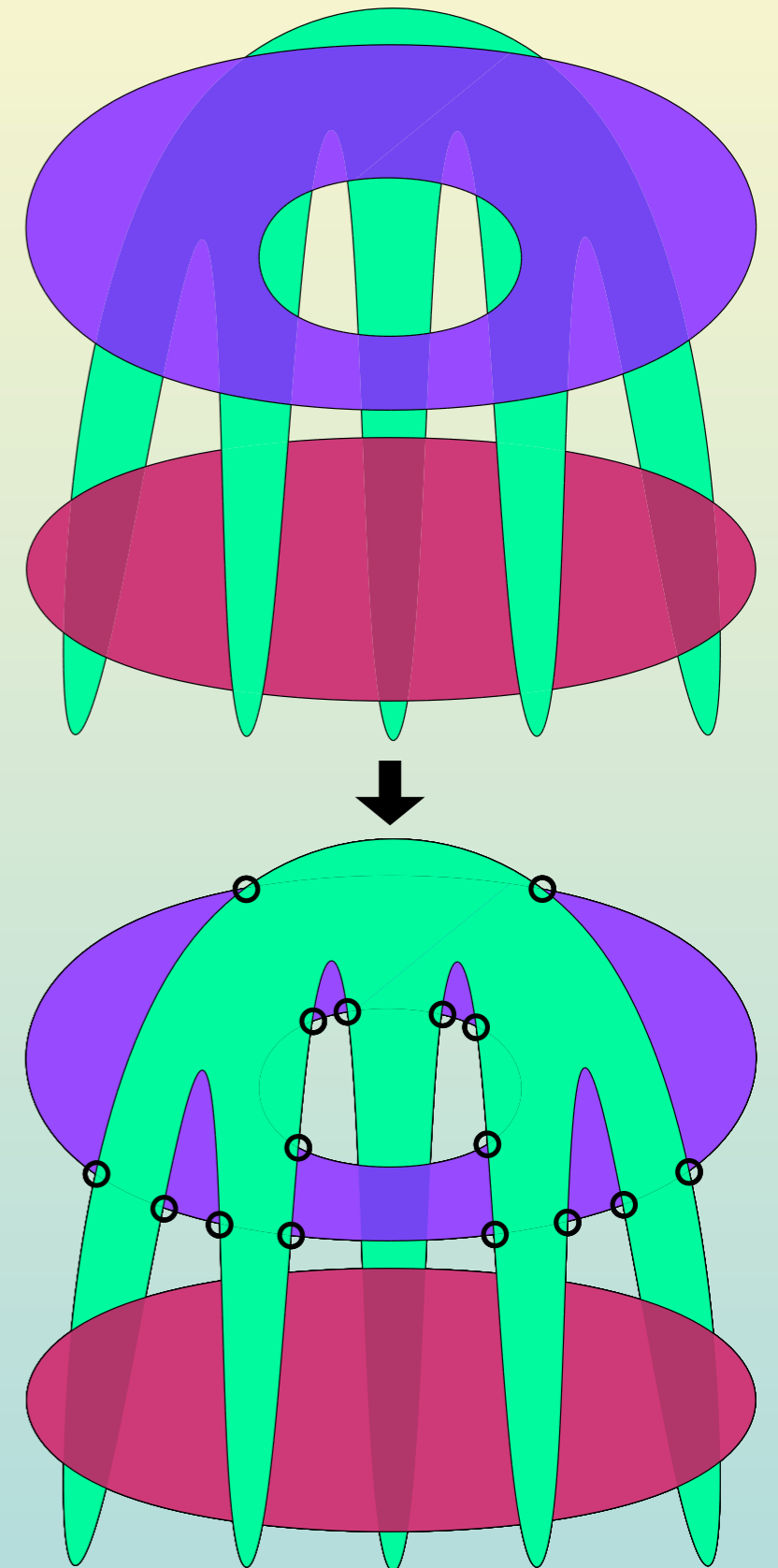
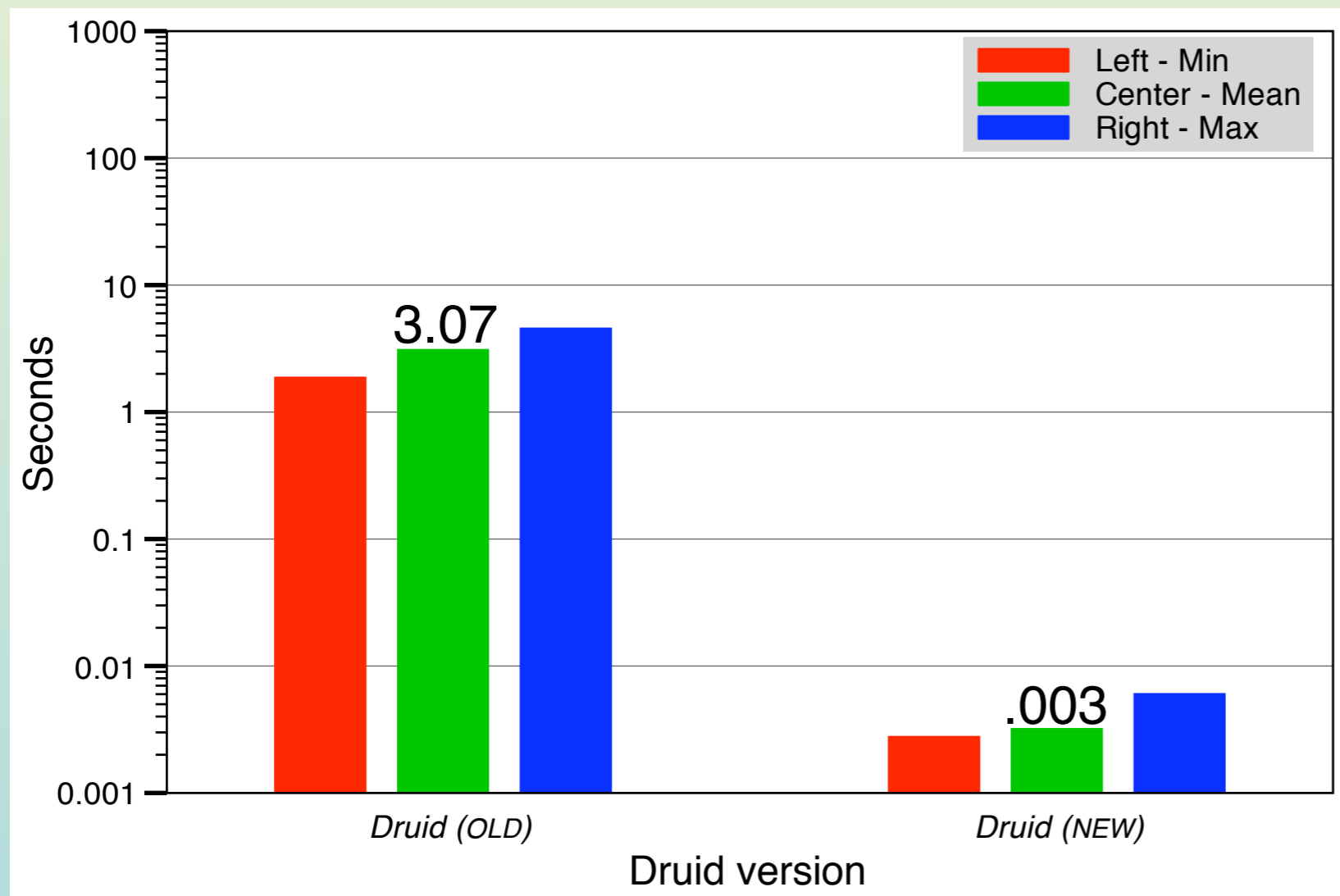


Min, mean, max with respect to a crossing-flip performed independently on each corner



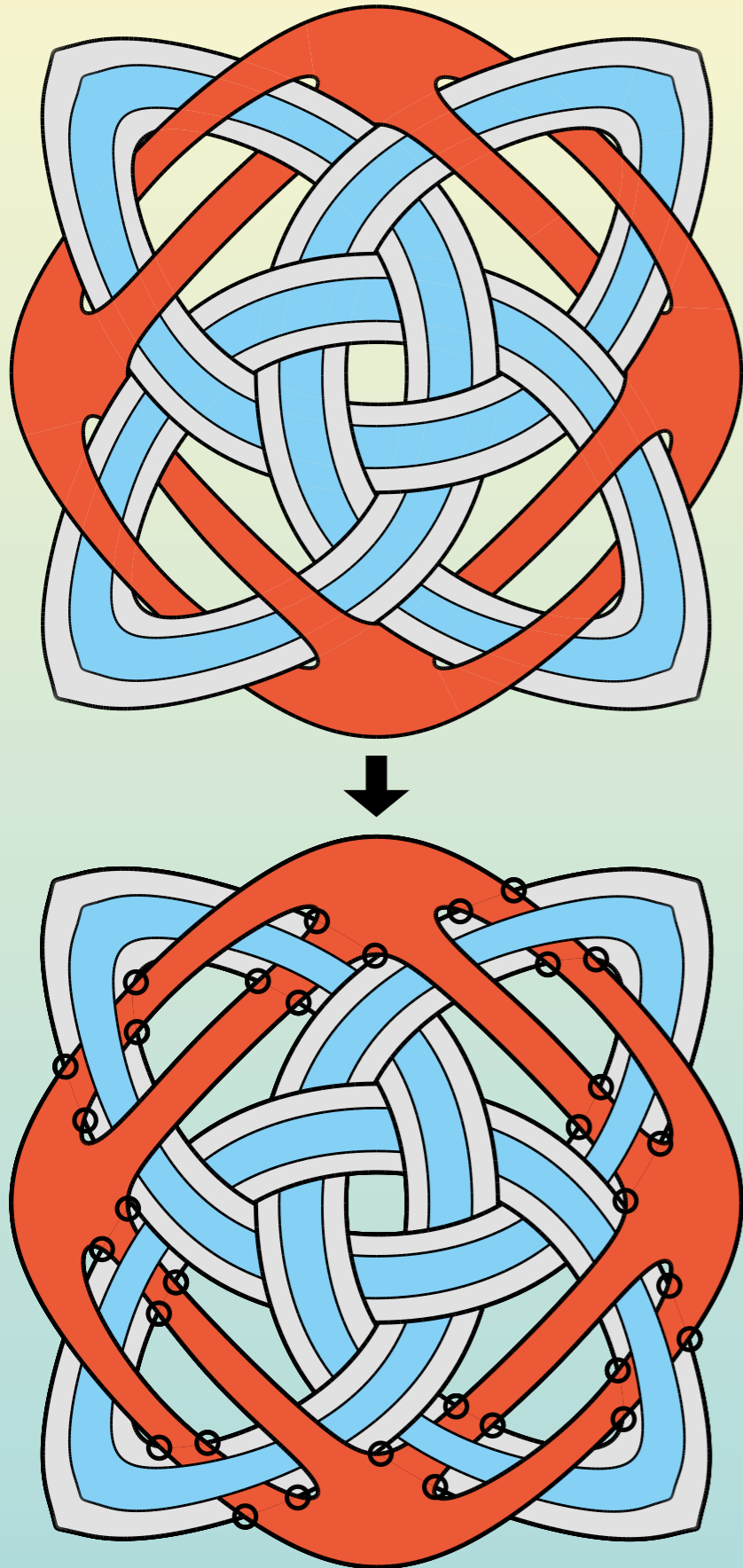
# Results: A Large CSEC Flip

- Size 16, indicated with circles
- *Druid (OLD)* cannot relabel in a reasonable time
- *Druid (NEW)* performs 967 times faster
- Note: *Druid (OLD)* failed 50% of the time

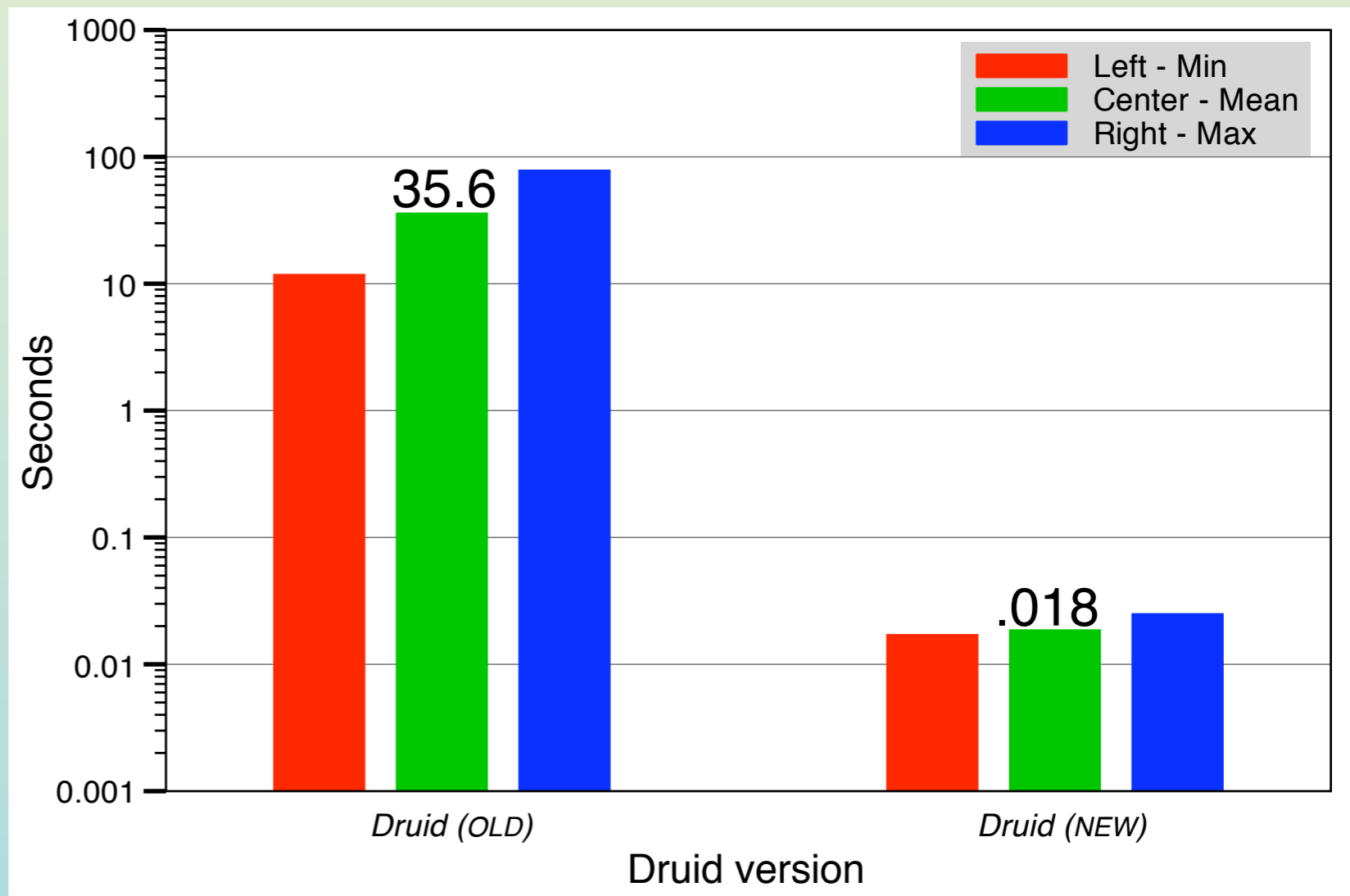


Min, mean, max with respect to a crossing-flip performed independently on each corner

# Results: A Complex Figure

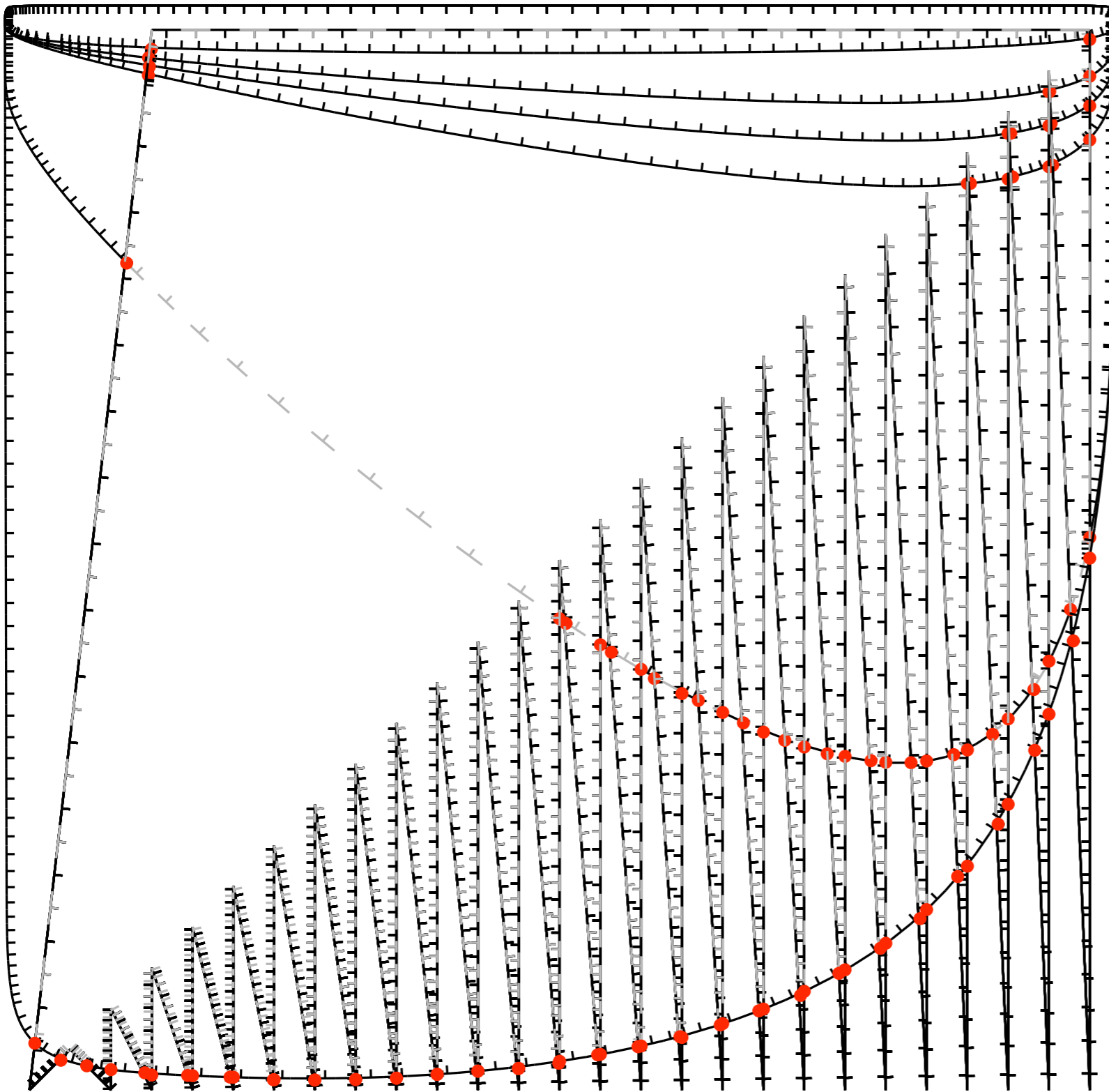


- 256 crossings, 64 CSECs
- *Druid (OLD)* cannot relabel this small CSEC flip in a reasonable time
- *Druid (NEW)* relabels in .02 seconds, 1900 times faster
- Note: *Druid (OLD)* failed 2% of the time





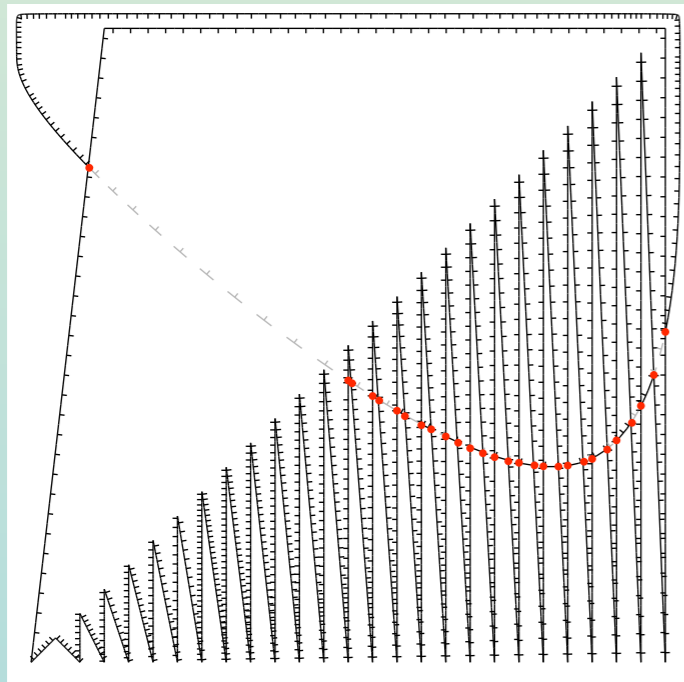
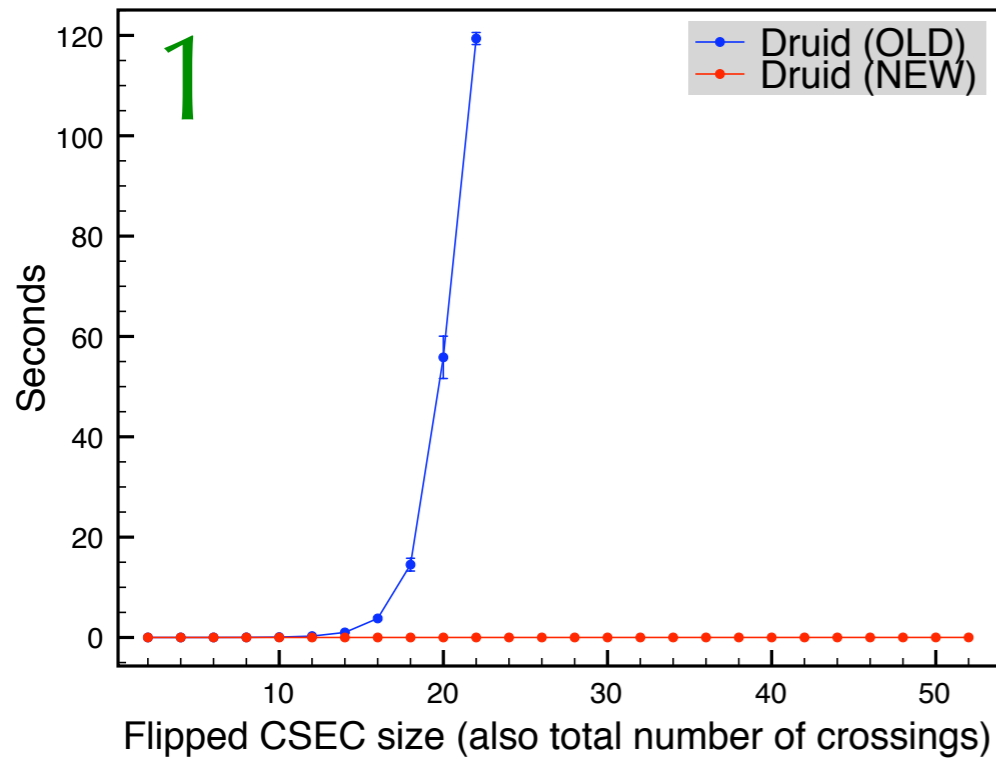
# CSEC Flip Performance



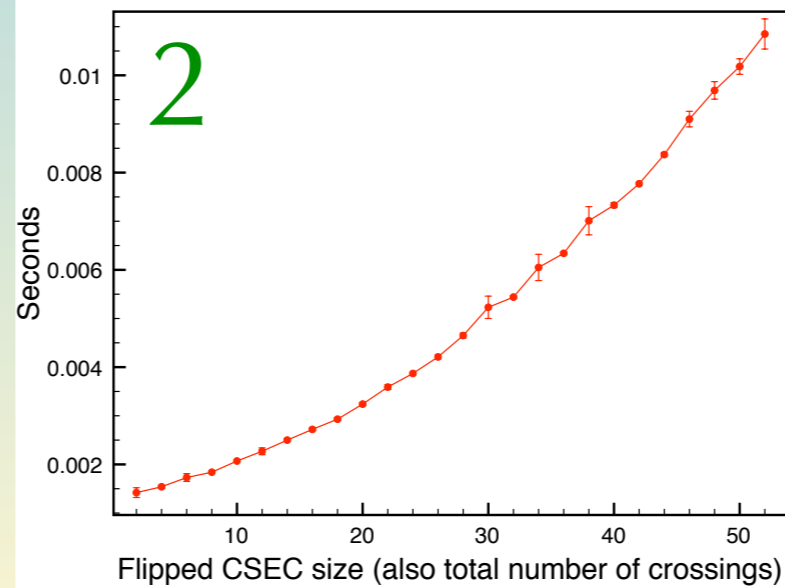
**Flipped CSEC size:**  
linear in the total  
number of crossings

# CSEC Flip Performance

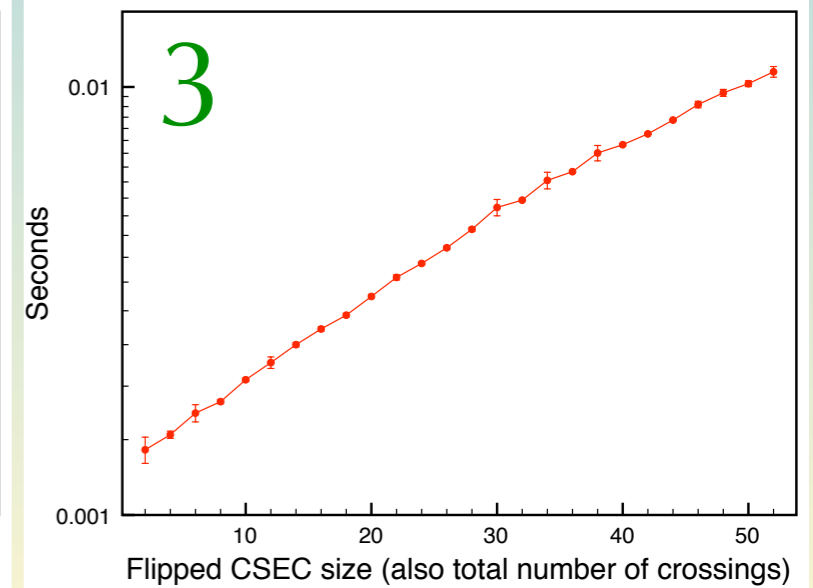
## Running time vs. CSEC size



CSEC Flip Performance (*Druid (NEW)*)

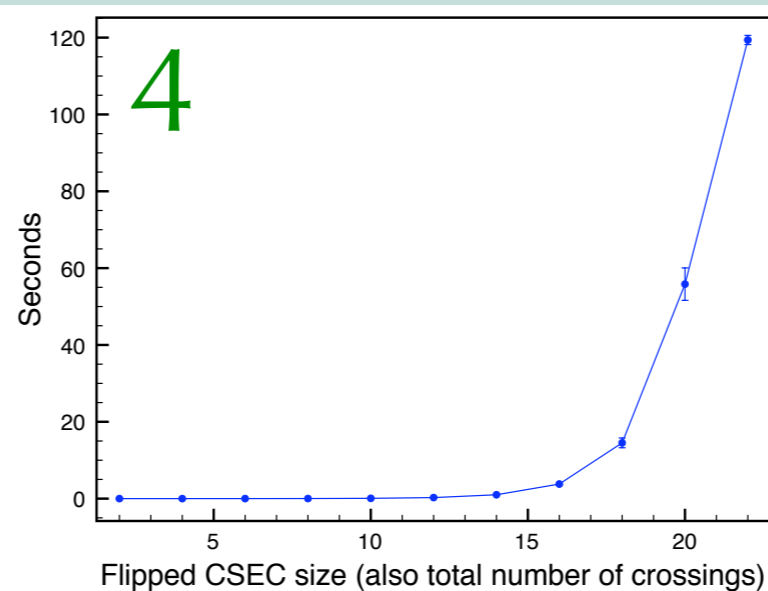


CSEC Flip Performance (*Druid (NEW)*)  
(log Y axis)

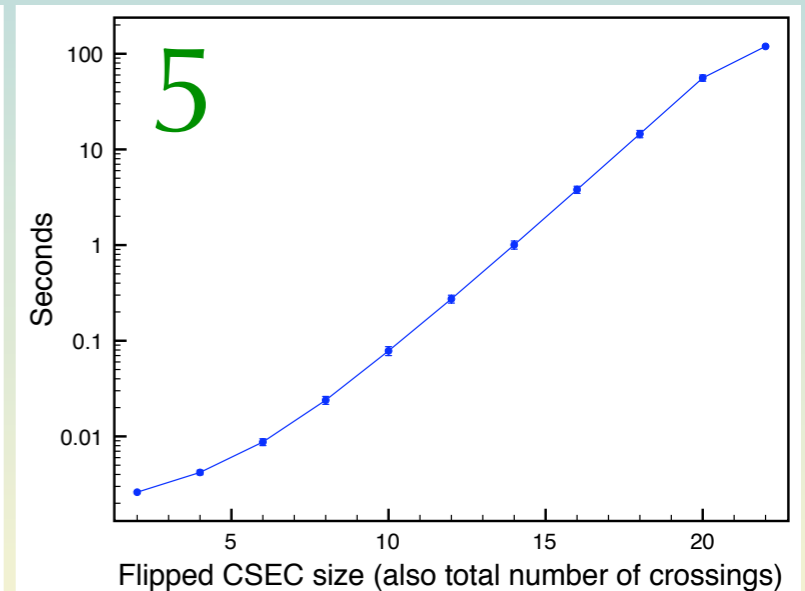


Performance is polynomial w.r.t. CSEC size

Search Performance (*Druid (OLD)*)



Search Performance (*Druid (OLD)*)  
(log Y axis)



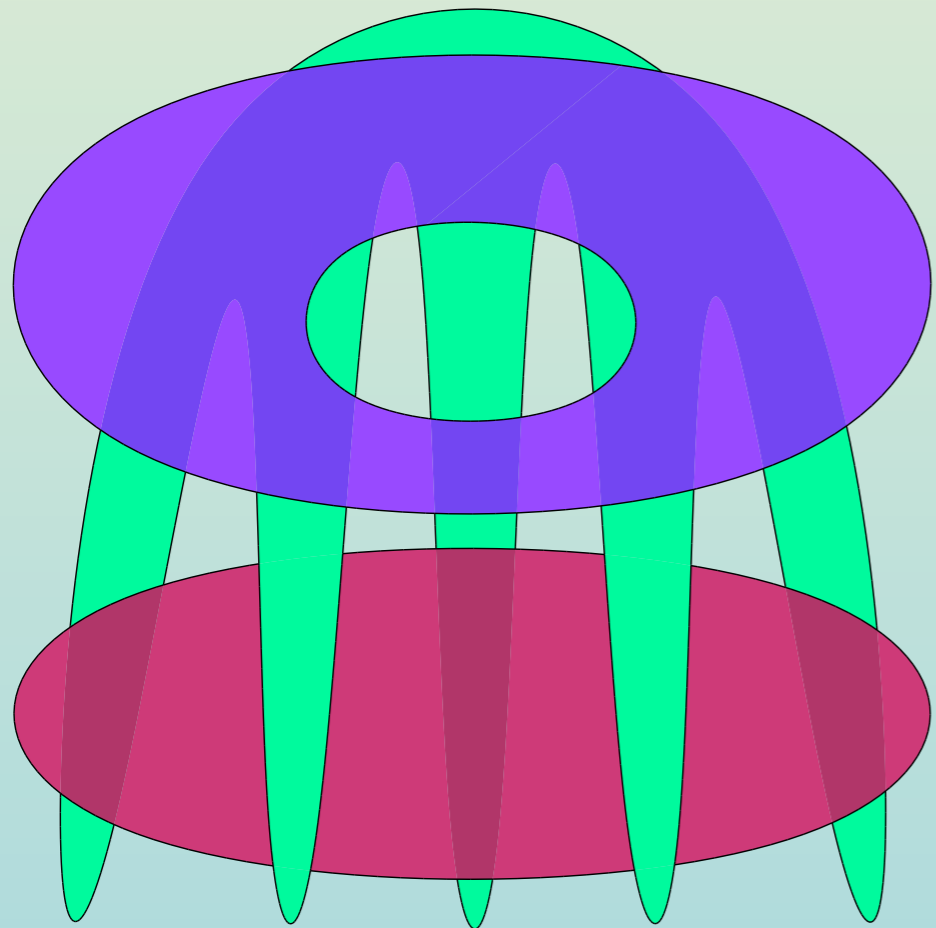
Performance is exponential w.r.t. CSEC size

# Future Work

- Labeling with CSECs
- *Locking* and *kinematic* interactions
- *Occluding contours* and *pita surfaces*

# Future Work: Labeling with CSECs

- CSECs have a profound effect on the search space size
- e.g., this drawing has 40 crossings but only 7 CSECs, an improvement by a factor of  $2^{33}$ , or 8.5 billion

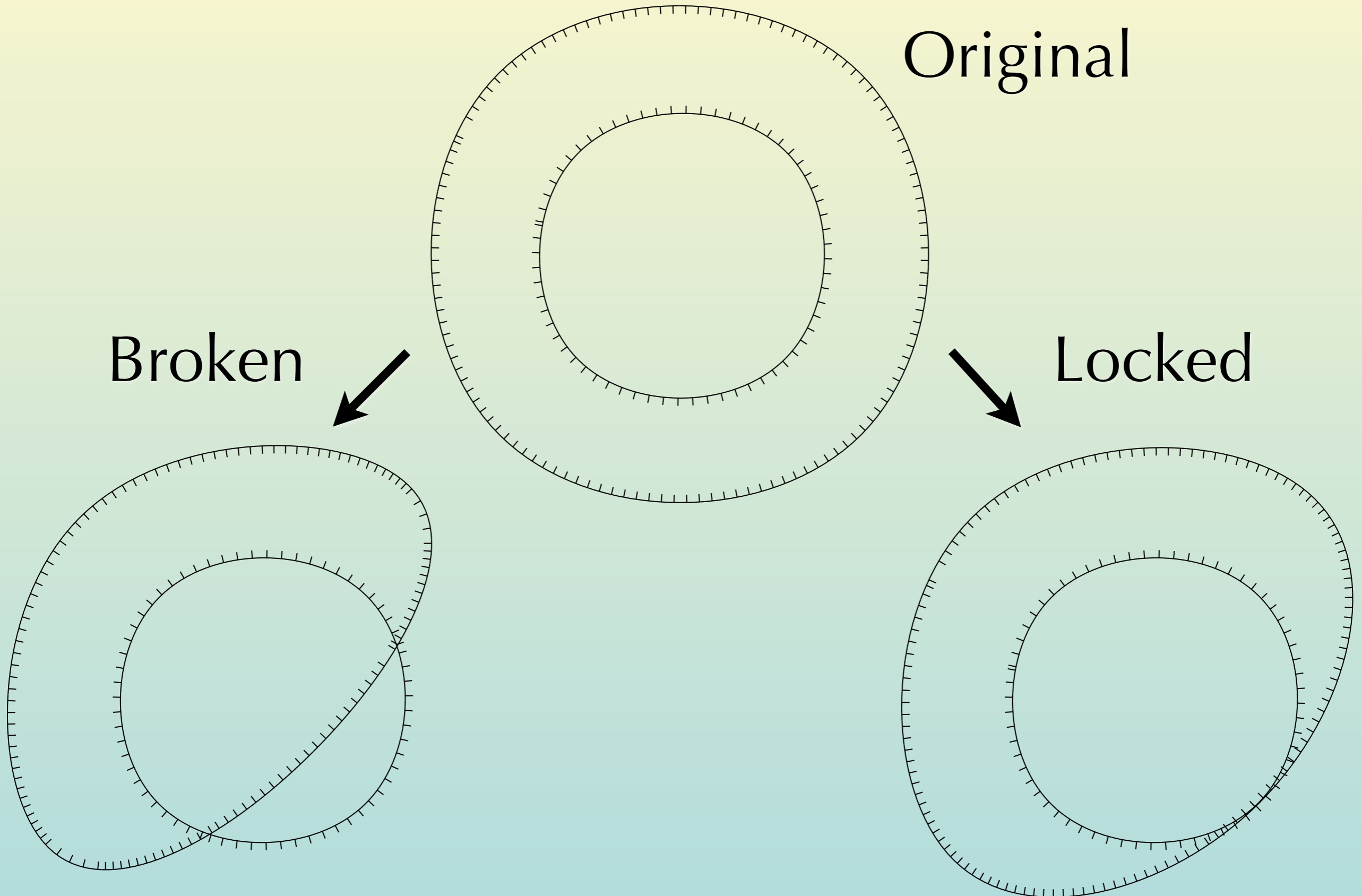


CSECs Used	Crossing-state search space size
No	$2^{40}$ (for 40 crossings)
Yes	$2^7$ (for 7 CSECs)

# Labeling with CSECs

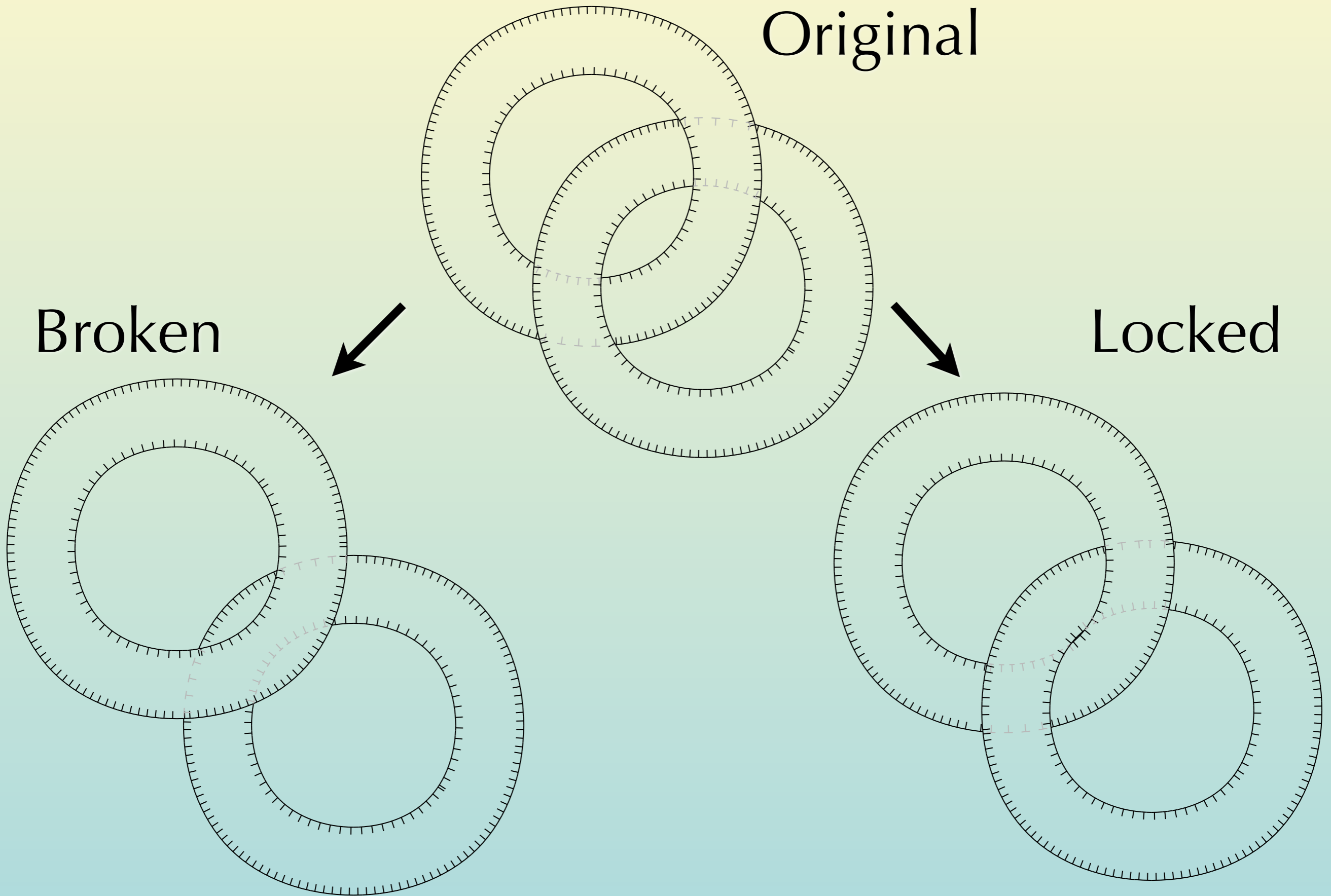
- Currently, can only find CSECs on legally labeled figures
- Cannot use CSECs to *label*, only to *relabel*
- Labeling must search the naive search space  $2^C$ , not the improved search space  $2^E$
- Having the CSECs for an unlabeled figure would greatly assist the labeling search

# Future Work: Locking Interactions





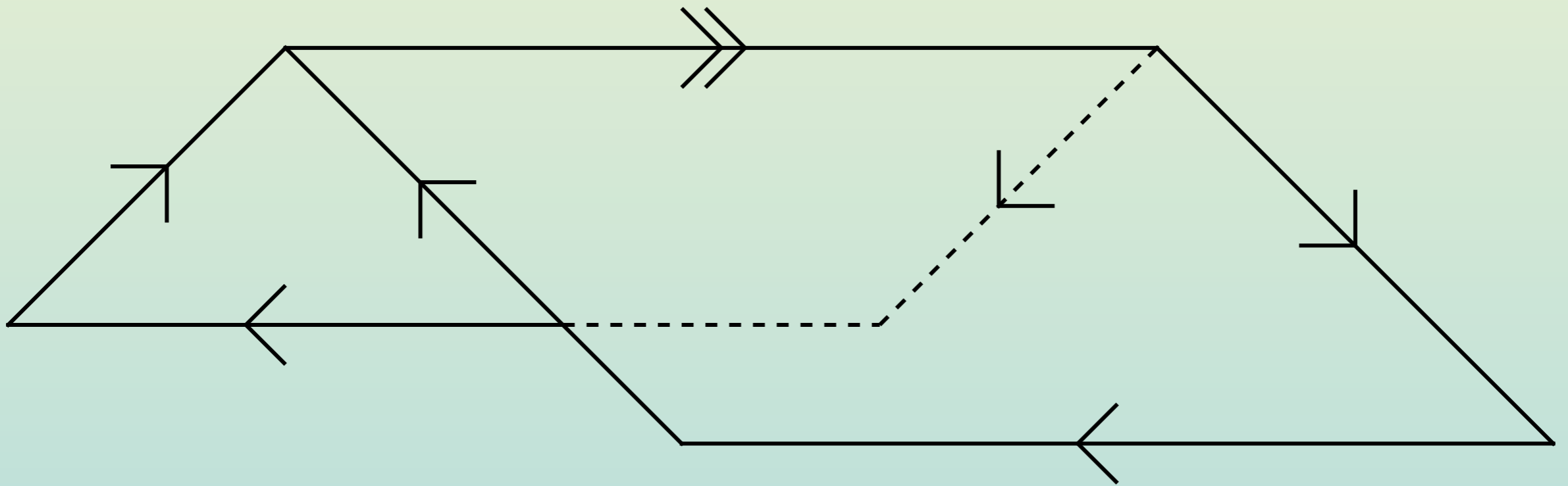
# Locking and Kinematic Interactions





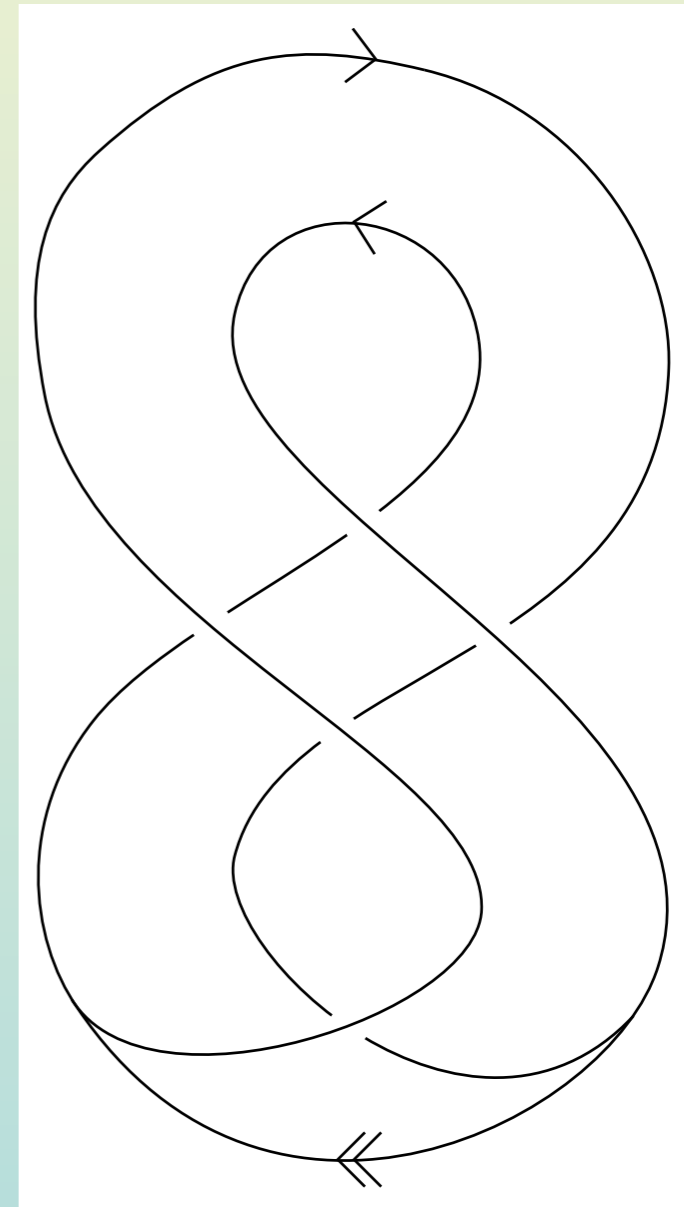
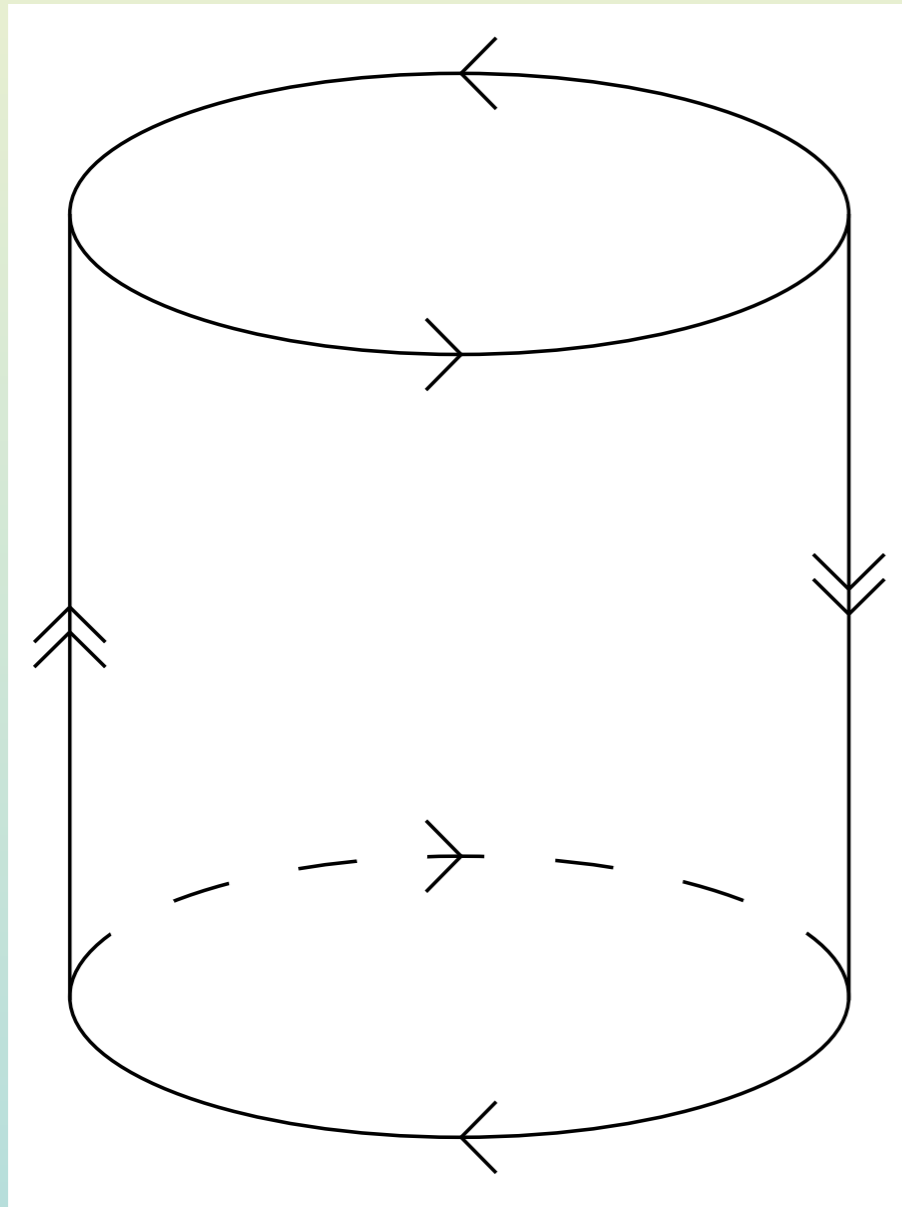
# Future Work: Occluding Contours and Pita Surfaces

An *occluding contour* is the projection of a fold.



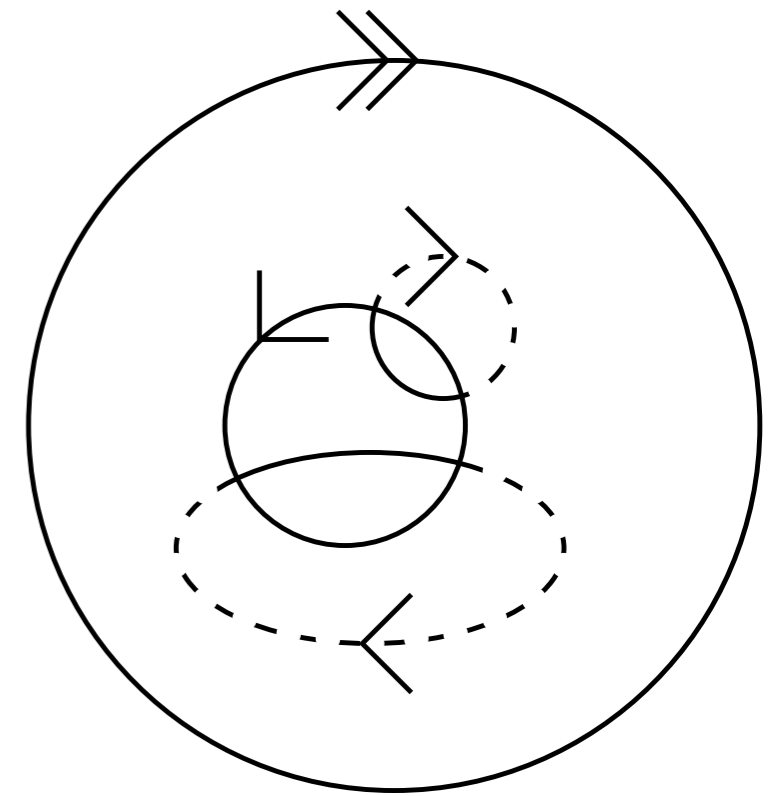
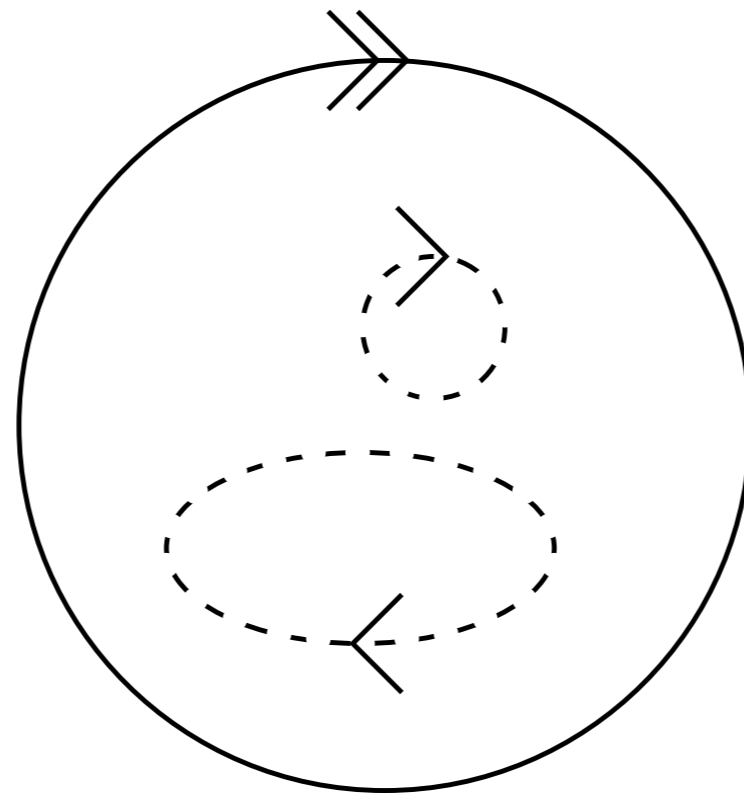
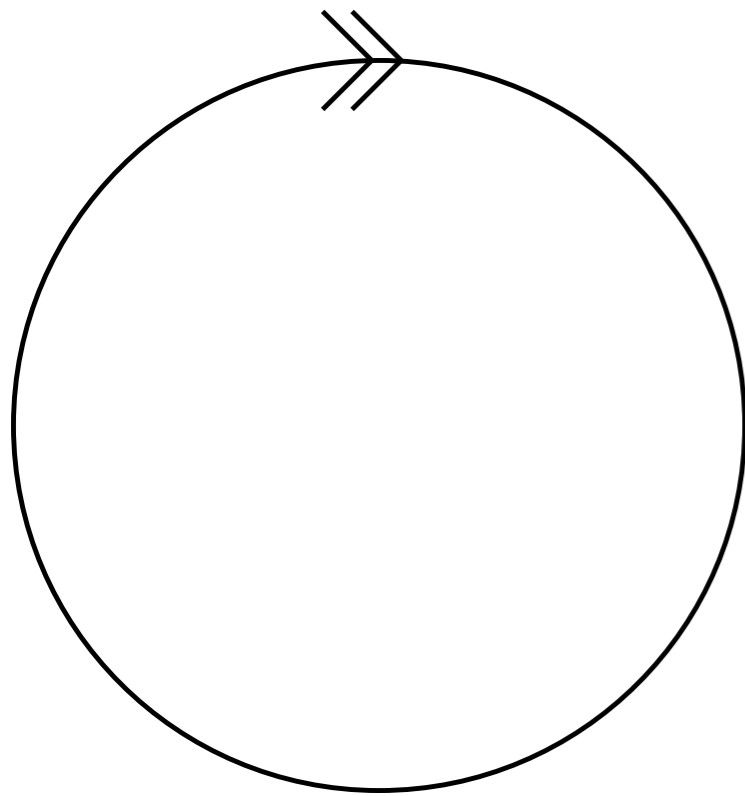
# Occluding Contours: Examples

Occluding contours enable construction of cylinders and Mobius strips.



# Occluding Contours: Pita Surfaces

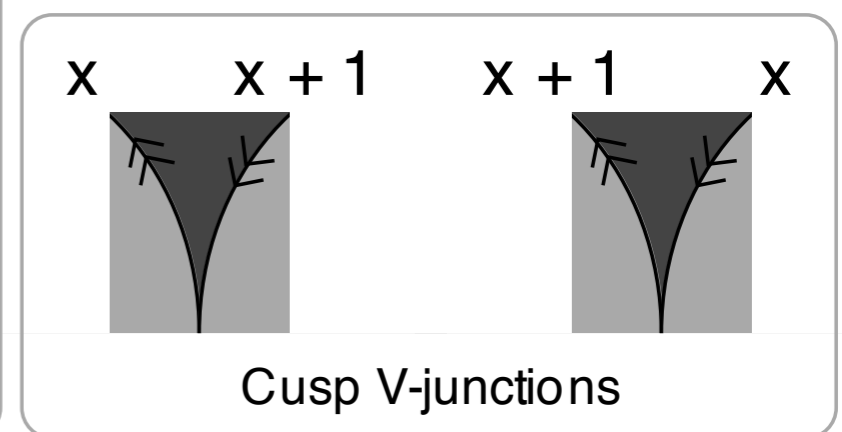
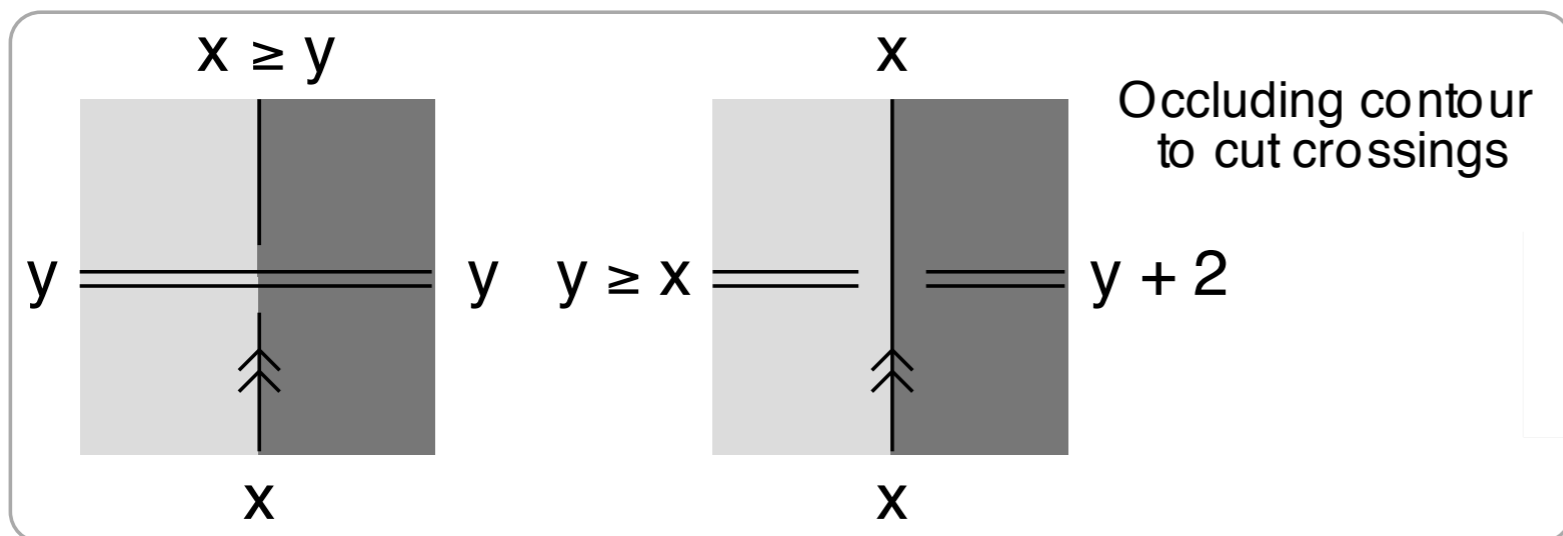
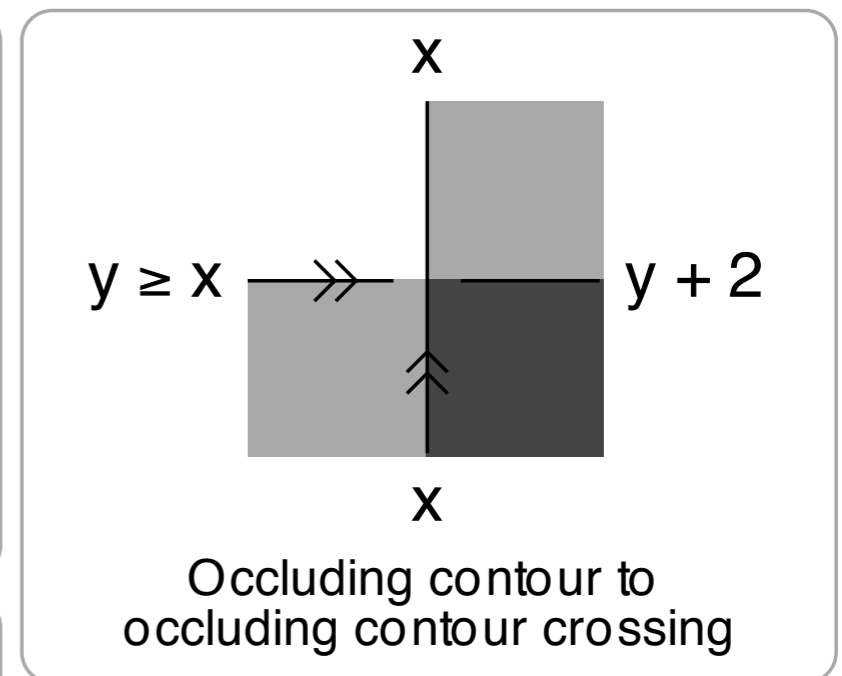
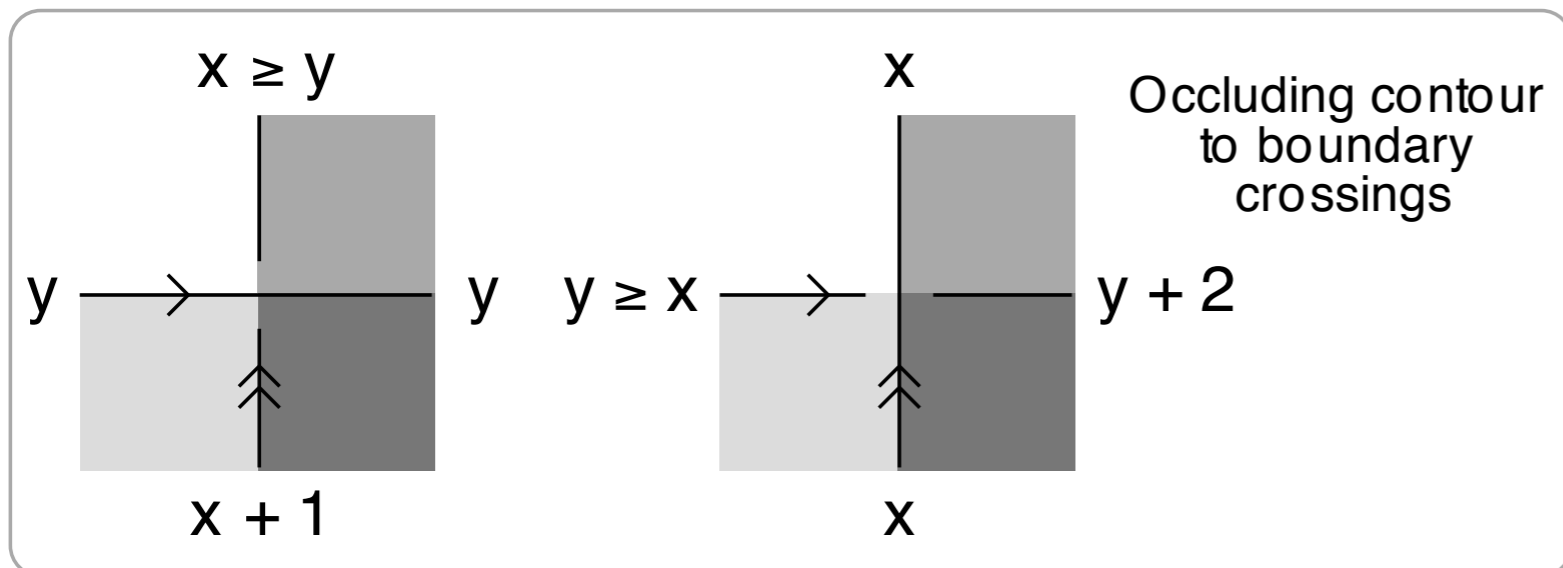
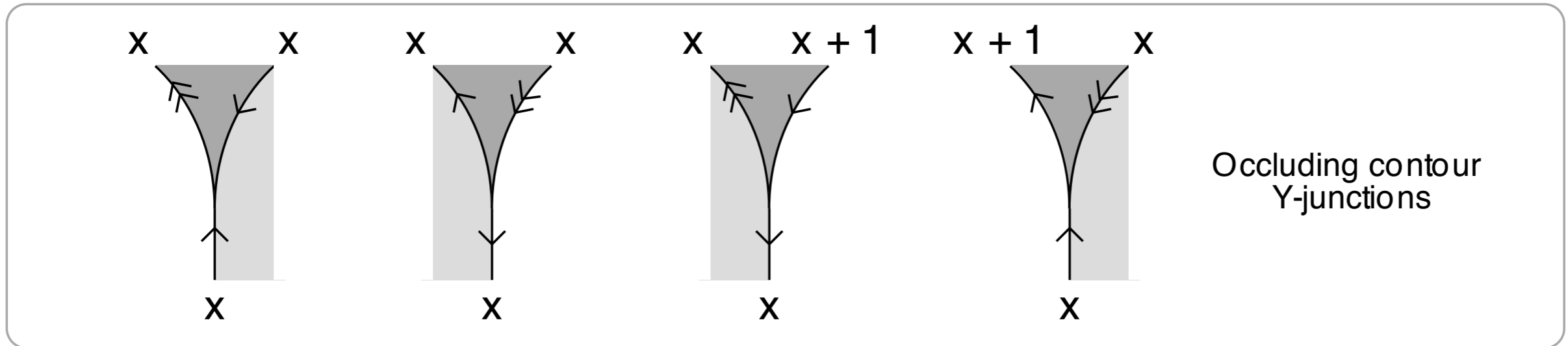
Occluding contours enable construction of *pita* surfaces.



pita surface

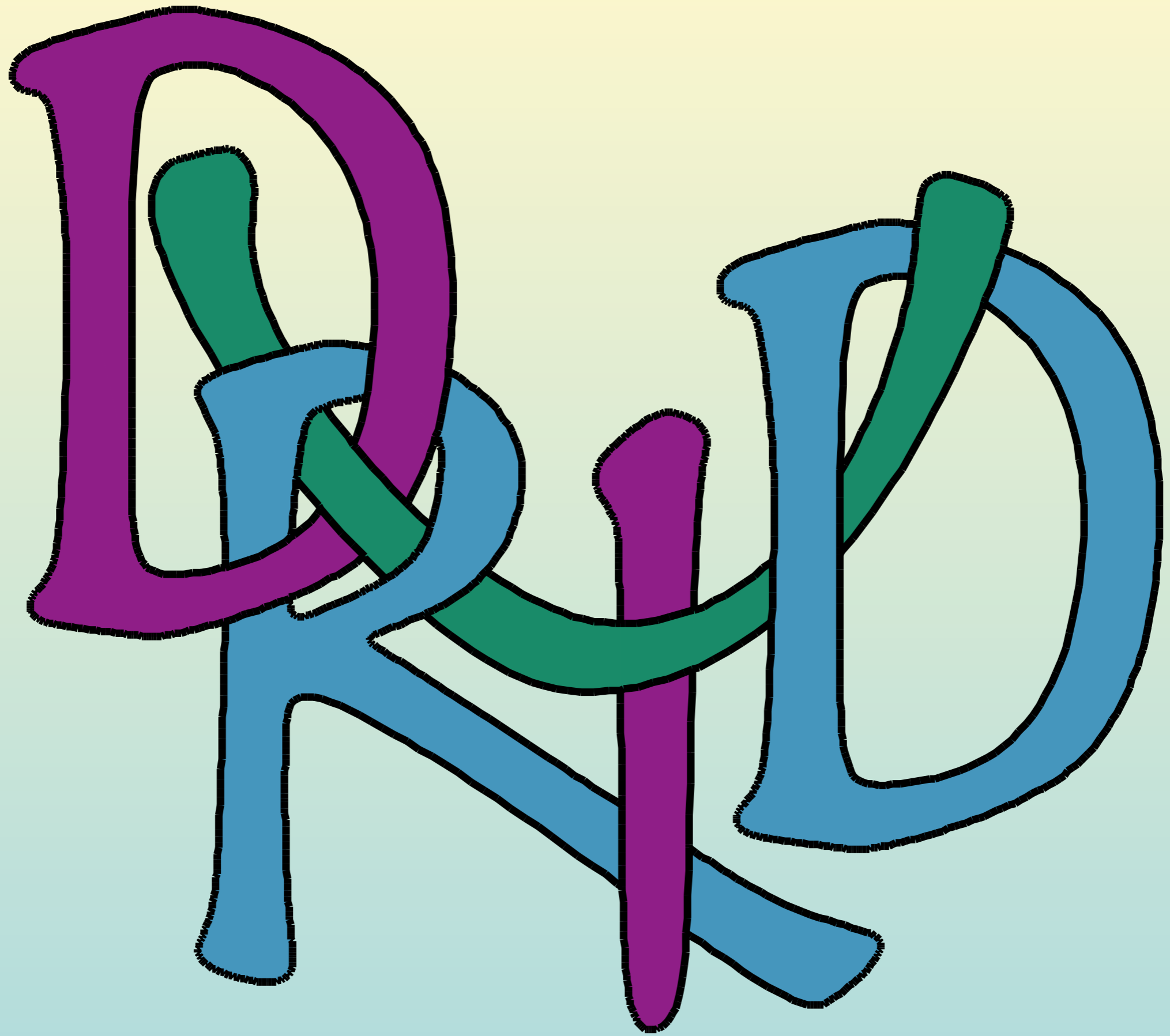
pita *containment*

# Occluding Contour Labeling Schemes



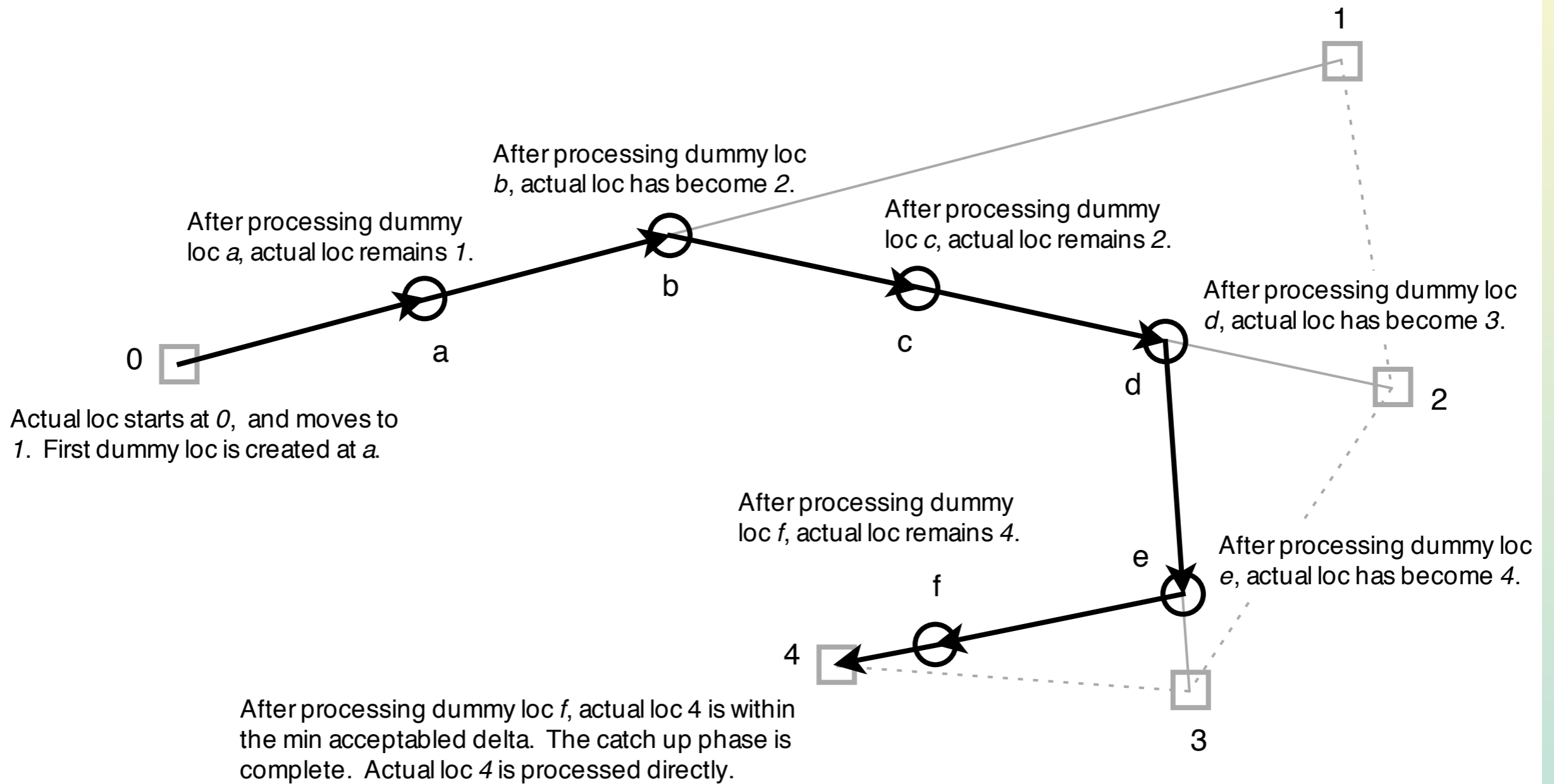
# Conclusions

- Developed *Druid*, a system for constructing interwoven 2½D scenes
- Use of branch-and-bound search to relabel gives the user the experience of interacting directly with idealized physical surfaces
- Search hinders *Druid's* scalability
- Discovered a topological property of 2½D scenes, the crossing-state equivalence class rule
- Exploitation of this property can alleviate the need to search in some situations, and can dramatically reduce the search space in remaining situations
- Vastly extended the complexity of drawings that users of *Druid* can construct





# Min. Acceptable Mouse Delta

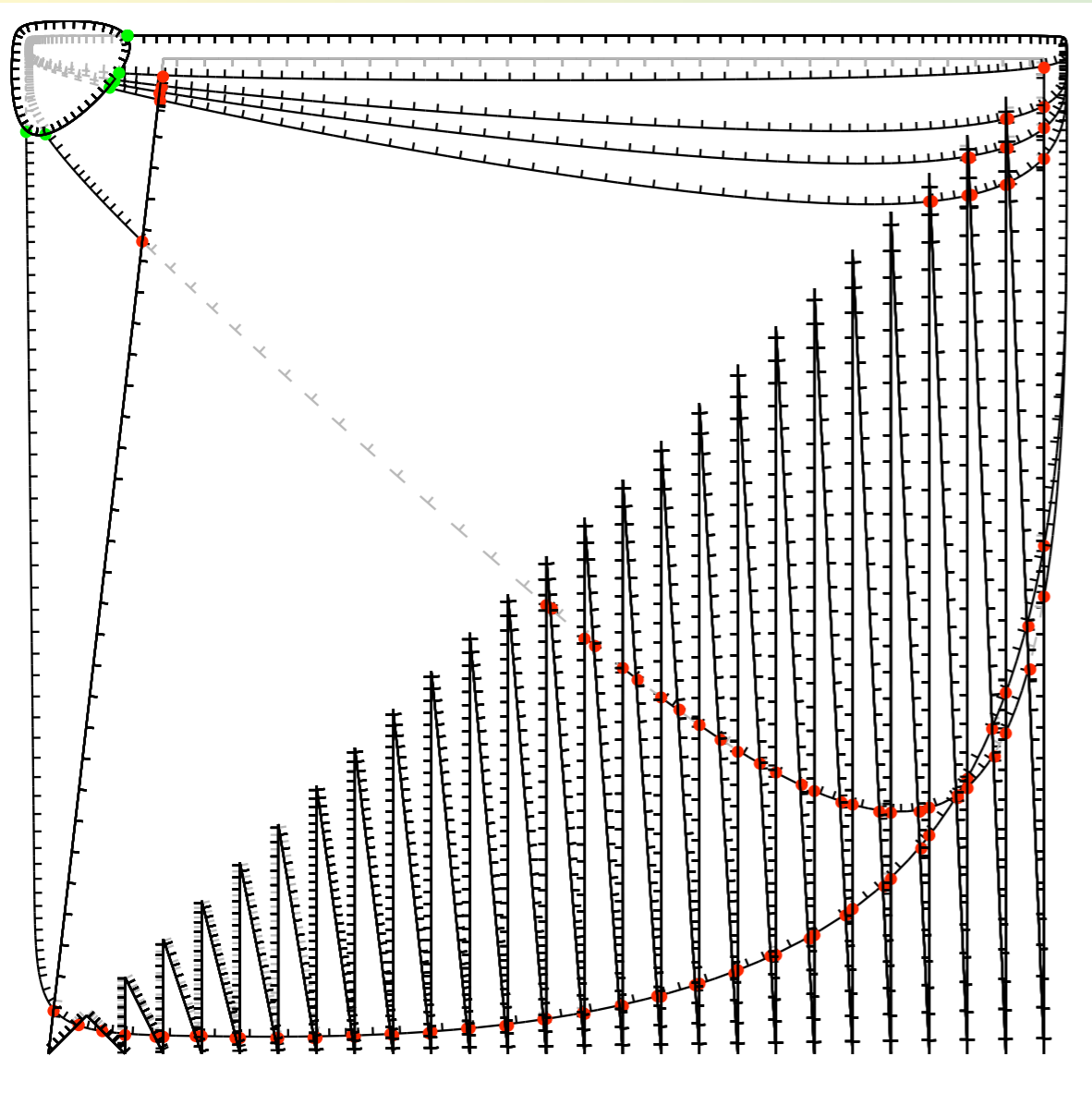


Minimum acceptable mouse delta

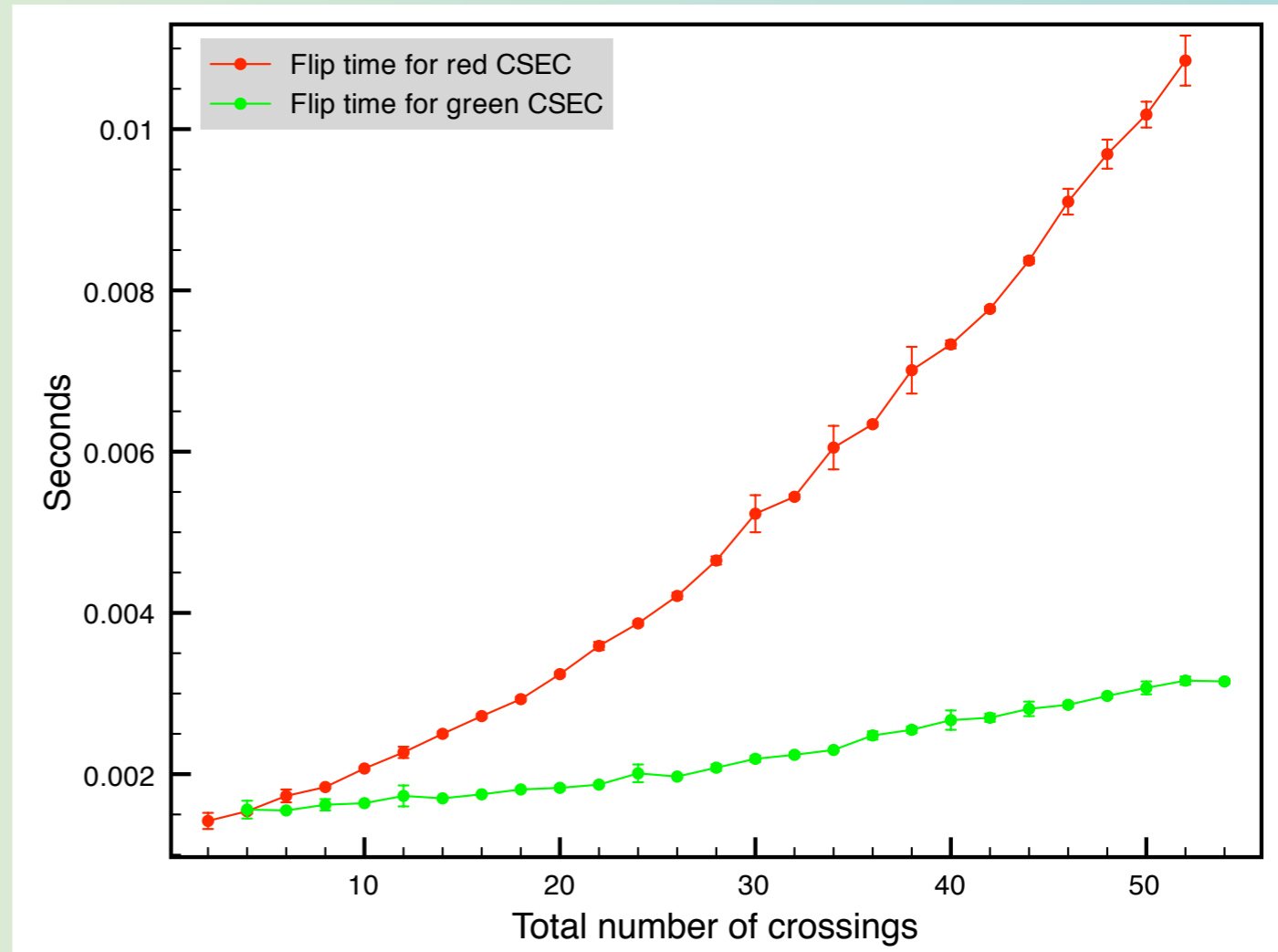
- Sequential actual mouse locations (numbers)
- Sequential dummy mouse locations (letters)
- Projection rays used to calculate dummy mouse locations
- ➔ Dummy mouse projections

# CSEC Flip Performance

Flipped CSEC size:  
constant (green)



Running time vs.  
Total Number of Crossings



Red plot is the same plot shown on the previous slide  
(seconds to perform the red CSEC flip)

# Depth Sort vs. *Druid*

## ● *Depth Sort:*

- Uses cuts to remove cycles and create a DAG.
- Renders by sorting polygons in 3D from back to front.

## ● *Druid:*

- Uses cuts to group boundaries **not** to remove cycles.
- Makes weaker assumptions to render than required by depth sort – does not require DAG.

[1] Angel, E. *Interactive Computer Graphics*. Addison-Wesley, 2006.

[2] Foley, vanDam, Feiner, and Hughes. *Computer Graphics, Principles and Practice*. Addison-Wesley, 2000.

# Scanline Algorithms vs. *Druid*

- ***Scanline algorithms:***

- Raster-based
- Method for rendering vector objects

- ***Druid:***

- Vector-based
- Relies on graphical API to render vector objects

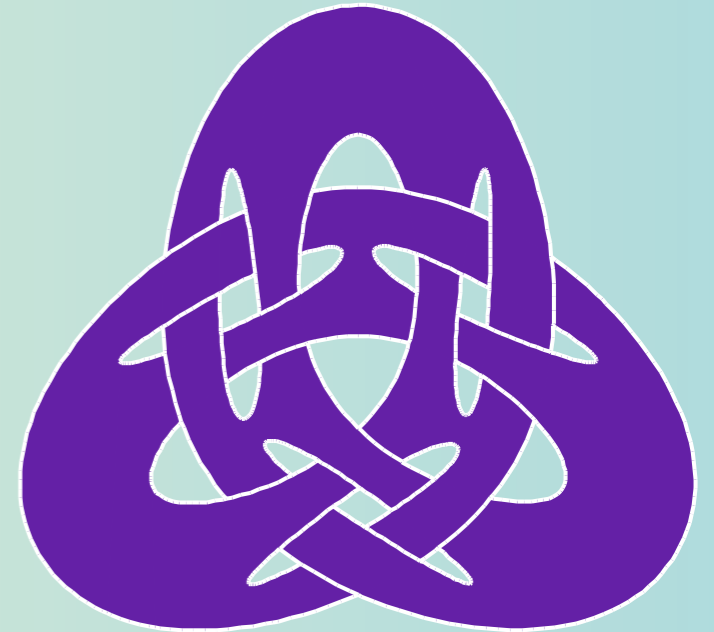
[1] Barkan, E., and D. Gordon. The Scanline Principle: Efficient Conversion of Display Algorithms into Scanline Mode. *The Visual Computer*, **15**(249), 1999.

[2] [http://www.devmaster.net/wiki/Scanline\\_algorithm](http://www.devmaster.net/wiki/Scanline_algorithm)

# Hidden Surface Removal vs. *Druid*

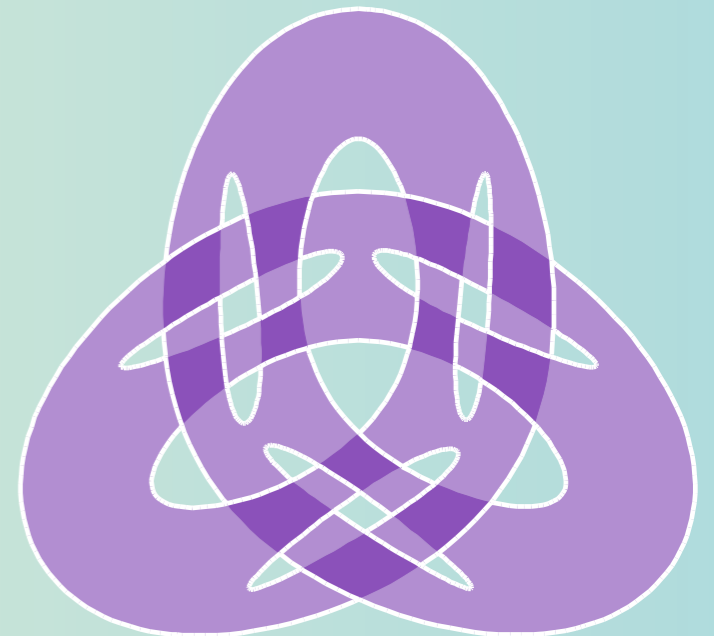
- ***Hidden surface removal:***

- Assumes opaque surfaces bounding solid objects



- ***Druid:***

- Assumes transparent fronto-parallel surfaces
- Opaque surfaces are a special case



[1] [Weiler K.](#) and [Atherton P.](#) Hidden Surface Removal Using Polygon Area Sorting. ACM SIGGRAPH Computer Graphics, *Proceedings of ACM SIGGRAPH 77*, **11**(3) pp. 214-222, 1977.

[2] Metelli, F., The perception of transparency, *Scientific American*, 230(4), pp. 90-98, 1974.